From single particle to correlation-dominated criticality in a level-crossing transition

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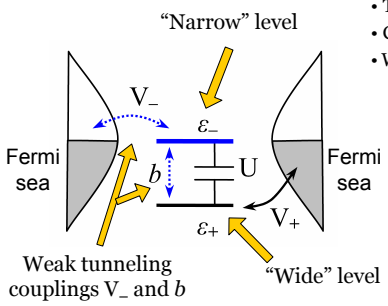
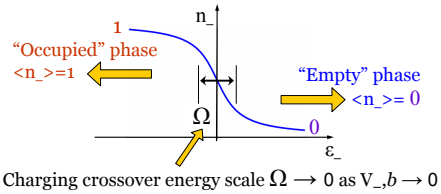
ESF EIROPAS SOCIĀLAIS FONDS

1 Model

- Single spinless level ("-")
- Tunneling directly to "-" band
- Tunneling via "+" level to another ("+") band
- Capacitative interaction $U n_+ n_-$
- Wide-band limit and zero temeprature



Study the narrow level limit $V_-, b \to 0$



Charging crossover energy scale $\Omega \to 0$ as $V_-, b \to 0$

$$\mathcal{H} = \sum_{\sigma=\pm} \epsilon_\sigma d_\sigma^\dagger d_\sigma + \frac{b}{2}\left(d_+^\dagger d_- + d_-^\dagger d_+\right) + \sum_{k,\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,\sigma} V_\sigma \left(c_{k\sigma}^\dagger d_\sigma + \text{H.c.}\right) + U n_+ n_-$$

2 Criticality

- Non-interacting case is trivial

$$\Omega = \Gamma_- \equiv \pi\rho|V_-|^2$$

- Claim:

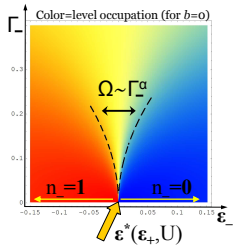
$$\boxed{\Omega \propto \Gamma_-^\alpha \qquad \Omega \propto b^{2\beta}}$$

(for b= 0 and V_=0 respectively)

- Single-parameter scaling:

$$\Omega(\Gamma_-, b) = A\Gamma_-^\alpha \, \mathfrak{F}\left(\frac{B b^{2\beta}}{A\Gamma_-^\alpha}\right)$$

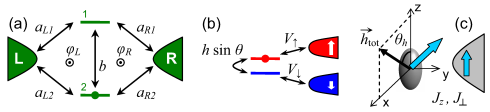
$$\mathfrak{F}(x) = \begin{cases} 1, & x \ll 1 \\ x, & x \gg 1 \end{cases}$$



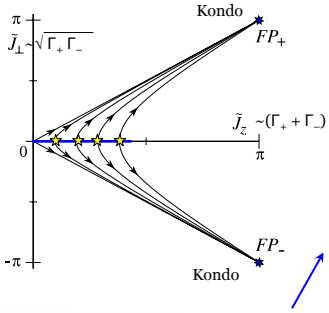
- "Quantum critical" point:
  a) A gapless degree of freedom decouples from the rest
  b) Degenerate ground state
  c) Singularity of the Green function at the Fermi surface
  d) Formally – not a Fermi liquid

4 Motivation & context

- Can think of "-" and "+" as isospi-½ projections
- In the isospin language the model is equivalent to a single spinful Anderson level with two generalizations:
  a) Different densities of spinup ("+") and spindown ("-") electrons
  ⇔ **ferromagnetic leads**
  b) Local Zeeman field in (z,x) plane = (e+-e-, b)
  ⇔ **spin index is not conserved for b ≠ 0**
- Any general (2 levels) x (2 leads) system **maps exactly onto this model [1]**



- Such few-level models with interaction has been studied intensively in the contest of population inversion and phase-lapses [2,3]
- It is our "critical manifold" $V_- = b = 0$ where the population inversion is expected [1] to be sharp [2]
- The model is directly related to
  a) charge sensing
  b) X-ray edge singularity
  c) non-Gaussian noise



5 More

- For large U, the vicinity of the charge inversion point maps [1] onto an anisotropic Kondo model
- Standard "poor man's" scaling equations [5] contain a critical line of unstable fixed points
- Characteristic energy scale is the Kondo temperature:

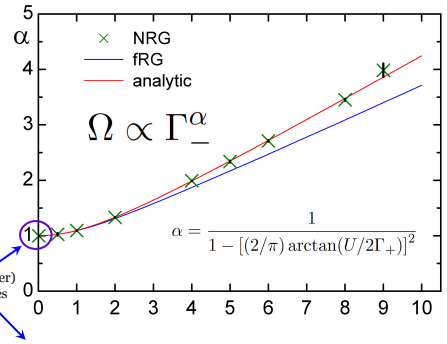
$$\Omega \propto T_K \propto \exp\left[\frac{\pi\epsilon(U+\epsilon)}{2U(\Gamma_+ - \Gamma_-)} \ln\frac{\Gamma_+}{\Gamma_-}\right] \overset{\Gamma_- \to 0}{\propto} \Gamma_-^{\pi U/(8\Gamma_+)} \implies \boxed{\alpha = \frac{\pi U}{8\Gamma_+}}$$

Correct asymptotics for NRG and X-ray results

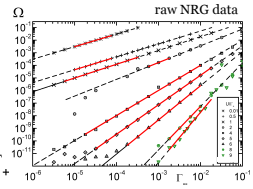
- We focused on $\epsilon_+ = -U/2$ case, but a general expression for the critical value of ϵ_- as a function of (U, ϵ_+) is

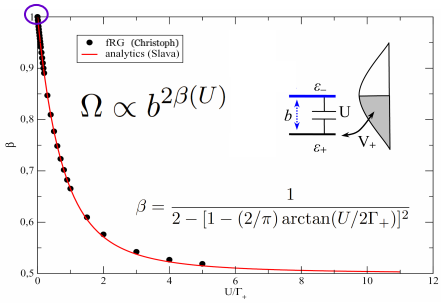
$$\epsilon^* \equiv -U/2 + \frac{1}{\pi}\left[(U+\epsilon_+)\arctan\frac{U+\epsilon_+}{\Gamma_+} - \epsilon_+ \arctan\frac{\epsilon_+}{\Gamma_+} + \frac{\Gamma_+}{2}\ln\frac{\epsilon_+^2+\Gamma_+^2}{(U+\epsilon_+)^2+\Gamma_+^2}\right]$$

3 Main results

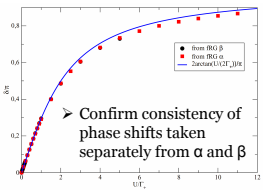


- Mapping to X-ray singularity problem [4] gives α, β in terms of phase shifts for "+" electrons
- Numerical Rernormalization Group diagonalizations confirm α(U) to better than 1% accuracy



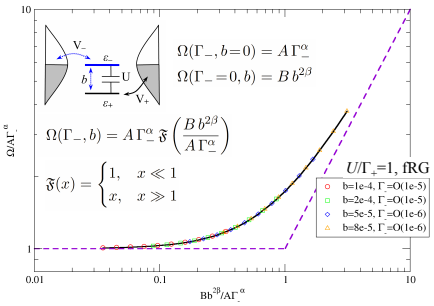


- Functional Renormalization Group offers a good approximation [2]
- Deviations at large U are expected (wrong prefactor in the exponent of Kondo scale)



- Confirm consistency of phase shifts taken separately from α and β

- Single parameter scaling confirmed by fRG



References

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