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Resonant transmission and quantized charge transfer in adiabatic **quantum pumping**

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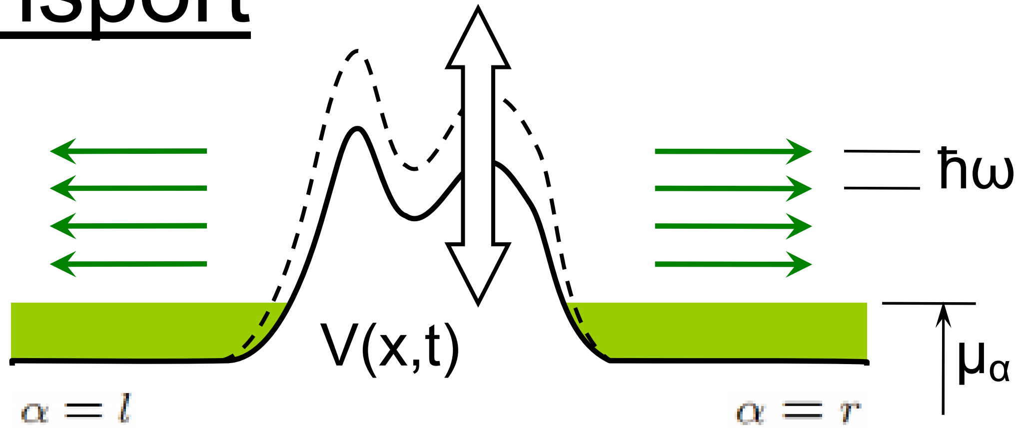
Theory of adiabatic transport



- Nanostructure connected to remote equilibrium reservoirs
- Coherent spinless electrons, no correlations
- External periodic potential acts locally:

$$V(x,t) = V(x, t + 2\pi/\omega)$$

- Consider t in $V(x,t)$ as a slow variable.
  Exact time-dependent scattering states are obtained from instantaneous ("frozen-t") energy eigenstates $\langle \chi_\alpha^t |$ as a perturbation series in powers of $\partial/\partial t$ [1].
- The instantaneous current from lead α in the leading (adiabatic) order is a sum of

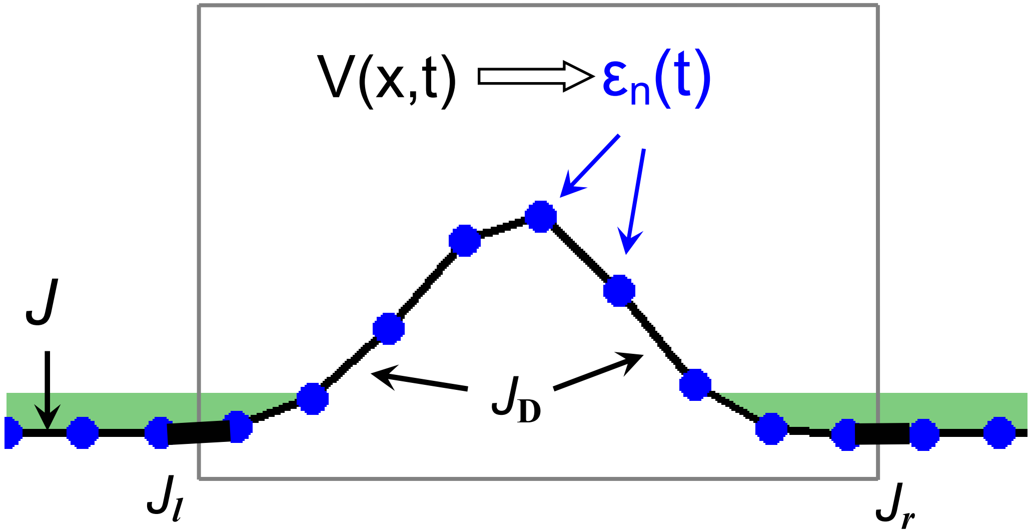
$$I_\alpha^{\text{pump}}(t) = \frac{e}{4\pi\hbar} \int dE \, \langle \chi_\alpha^t | \dot{V} | \chi_\alpha^t \rangle \frac{\partial (f_l + f_r)}{\partial E}$$ (equivalent of Brouwer f-la [2])

$$I_\alpha^{\text{bias}}(t) = \frac{e}{2\pi\hbar} \int dE \left\{ (f_l - f_r)\mathcal{T} + \frac{\hbar}{2} \frac{\partial (f_l - f_r)}{\partial E} \mathcal{T} \dot{\theta} \right\}$$ (equivalent of Landauer f-la)

  Here $\mathcal{T}$ *is the probability and θ is the phase of instantaneous transmission,* $f(E)$ *is Fermi distr.*

- Adiabaticity holds as long as ħω << typical energy scale of $\mathcal{T}(E)$ variation (e.g., resonance width Γ) [1,7]

Requires general solution of the static scattering problem at every t



Calculation method

- Discretize external potential on N points (tight-binding sites)
- On-site energies

$$\varepsilon_n(t) \equiv V(na, t)$$

- Nearest-neighbour hoppings J describe the kinetic energy:

$$\mu = -2J \cos k_F a$$

Charge transferred per one period (zero temperature and zero-bias) [3]:

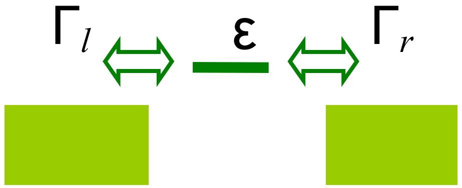
$$Q_\alpha \equiv \int_0^{2\pi/\omega} I_\alpha^{\text{pump}}(t)\, dt = \frac{e\, J_\alpha \sin k_F a}{\pi\, J} \int \sum_{n=1}^{N} |g_{n,1}|^2\, \dot{\varepsilon}_n\, dt$$

- Integrand is an inverse of a N × N matirx (Green's function for fixed t and E)
- If $V(x,t)$ is a polynomial in $\sin \omega t$ and $\cos \omega t$, the integration is done analytically

Gives the adiabatically pumped charge Q, once V(x,t) is specified

Resonance approximation

A weakly coupled state shows a Breit-Wigner resonance in



transmission

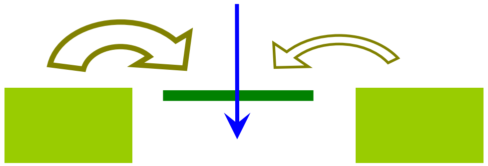
$$\mathcal{T} = \frac{\Gamma_l \Gamma_r}{(\mu - \varepsilon)^2 + (\Gamma_l + \Gamma_r)^2/4}$$

pumping current [4]

$$I_\alpha = \frac{e}{2\pi} \frac{-\Gamma_\alpha \dot{\varepsilon} + \dot{\Gamma}_\alpha (\mu - \varepsilon)}{(\mu - \varepsilon)^2 + (\Gamma_l + \Gamma_r)^2/4}$$

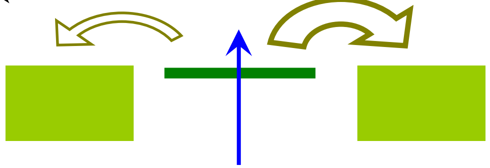
If the couplings vary slowly, the Lorentzian integrates to 1:

$$\Delta Q_\alpha \equiv \int_{t_1}^{t_2} I_\alpha \, dt \approx \pm \frac{\Gamma_\alpha}{\Gamma_l + \Gamma_r} \times e$$



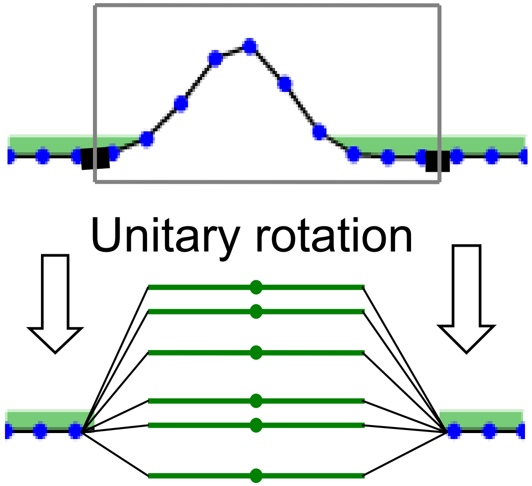
$\Delta Q_\alpha > 0$

Loading, dε/dt < 0



$\Delta Q_\alpha < 0$

Unloading, dε/dt > 0

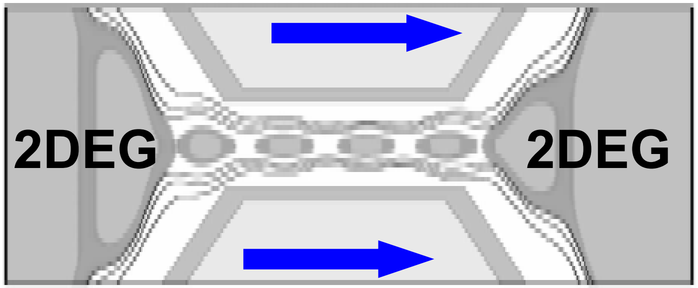


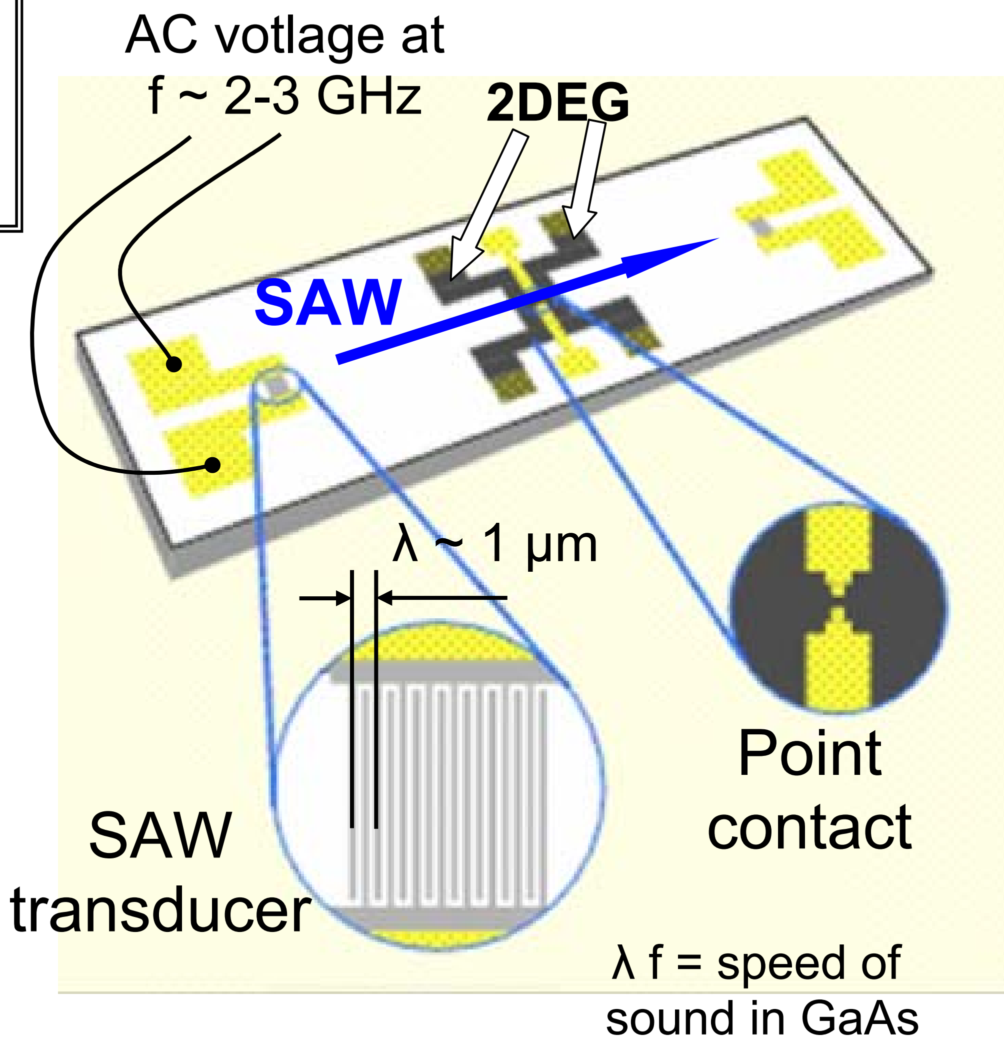
Can be generalized to several states

Approximates Q as a sum of loading/unloading contributions

Quantized pumping with **S**urface **A**coustic **W**aves

- A running wave of mechanical deformation creates a **moving potential profile** due to piezoelectric properties of GaAs
- In the depleted region of a point contact **screening is reduced**
- Periodic potential can capture and transfer an integer number of electrons





- Quantized transport achieved in experiments by V.I.Talyanskii *et al*. (Cambridge, UK, 1996 – ...) [5]

What can we learn from an idealized adiabatic pumping model?

A simple model

- 1D geometry (single transverse mode)
- Assume complete screening outside a region of length L
- Take the simplest form for the potential induced by the gates and the SAW[3,6]:

Gate voltage defines a static barrier

Optional gradual screening

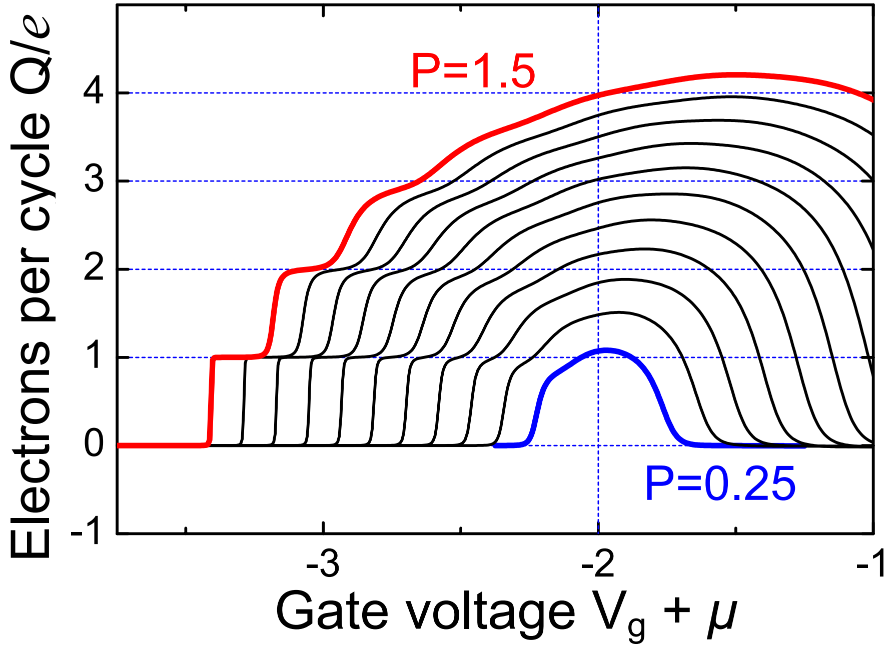
$$V(x,t) = \left[-V_g + P\cos(qx - \omega t)\right] \times e^{-x^2/L_s^2}$$

SAW amplitude

SAW wave-vector $q = 2\pi/\lambda$

- Charge transfer Q per period (DC component of the acoustoelectric current) is calculated using the **theory of adiabatic pumping** [1,3] (see details on the left)

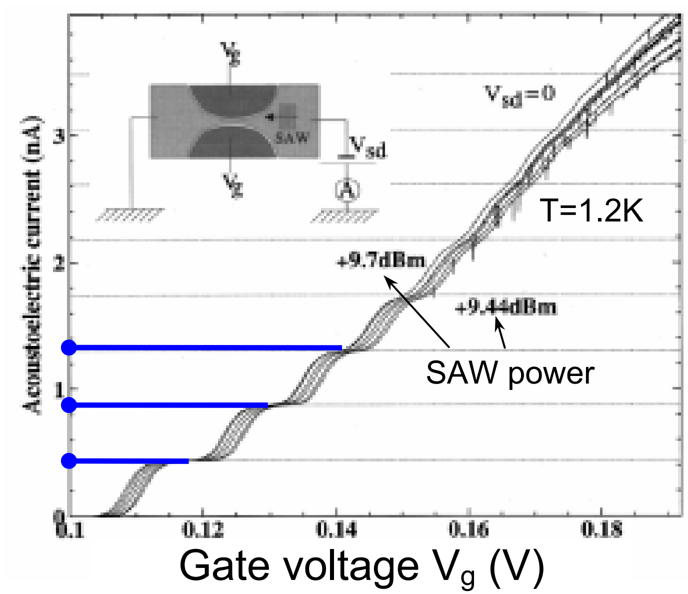
Calculation outcome



Main experimental features are reproduced:

- quantization of the acoustoelectirc current below conduction pinch-off
- greater SAW amplitude results in more steps
- higher steps are less accurate

Acoustoelectirc current =
= frequency × (1, 2, 3, …) × e



Experimental data from [5]

Model parameters:

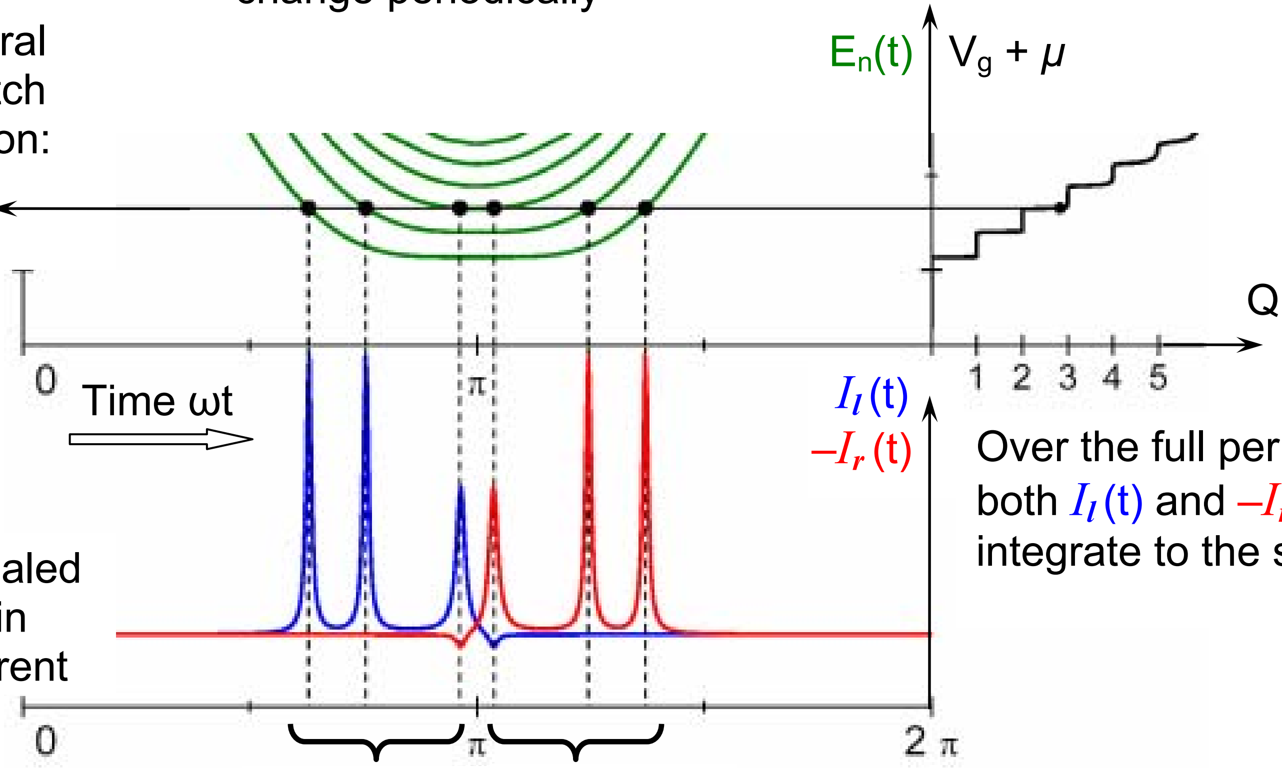
$\lambda = 2$ L ; $L_s = 4$ L
J = 1 (sets the unit of energy)
$k_F a = \pi/12$ (like free electrons)
N ≡ L / a = 24 sites

How does the staircase form?

Eigenenergies $E_n(t)$ of an isolated channel (no gates, no leads) change periodically

As time evolves, several energy levels can match the resonance condition:

$E_n(t) = V_g + \mu$



Over the full period, both I_l(t) and $-I_r$(t) integrate to the same Q

Each matching is signaled by a resonance peak in the instantaneous current

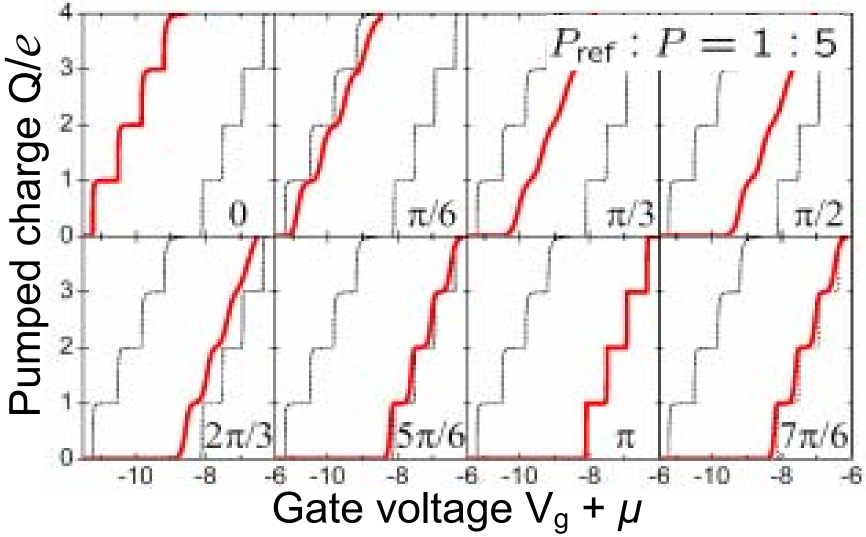
In the first half-period, the minimum of V(x) is on the left → $\Gamma_l \gg \Gamma_r$

In the second half-period, the captured electrons are more likely to unload to the right, $\Gamma_l \ll \Gamma_r$

Resonance picture builds a bridge towards the 'moving QD' scenario

Secondary SAW effects

$$V(x,t) = -V_g + P\cos(qx - \omega t) + P_{\mathrm{ref}}\cos(qx + \omega t + \varphi)$$



The sharpest steps are observed for φ=0 and π

Model parameters: P =8 J; λ = 4 L ; L_s = ∞; N = 10

- A weak counter-propagating wave due to reflection or a second SAW transducer
- Slope of the first step at Q=e/2 [6]

$$\frac{dQ}{dV_g} \approx \frac{A}{B + qL\,P_{\mathrm{ref}}\sin\varphi} \quad (A,B=\text{const})$$

In a left-right symmetric channel B =0

- Tuning the denominator to zero gives the sharpest staircase
- Dramatic increase in quantization accuracy for specifically tuned P_{ref} and φ has been observed experimentally [5]

What have we learned?

The SAW model example demonstrates a set of tools to study adiabatic quantum pumping due to complex potentials:

- Exact calculation reproduces the detailed qualitative features of the experiment
- Analysis of the model through the lens of the resonance approximation confirms the "moving quantum dot'' scenario
- Two major obstacles on the way to reliable quantitative predictions: interactions and non-adiabaticity

References

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[2] P. W. Brouwer, Phys. Rev. B **58**, 10 135 (1998).
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[5] J. Cunningham, V. I. Talyanskii, J. M. Shilton, M. Pepper, A. Kristensen, and P. E. Lindelof, Phys. Rev. B **62**, 1564 (2000), contains references to earlier experimental work.
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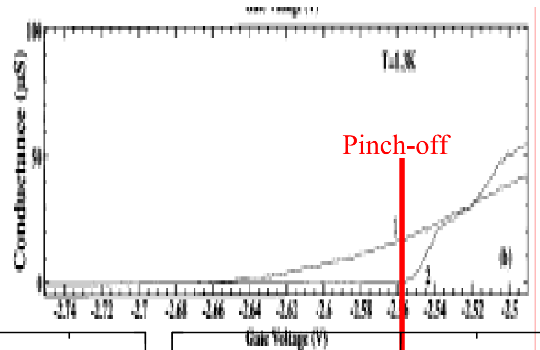
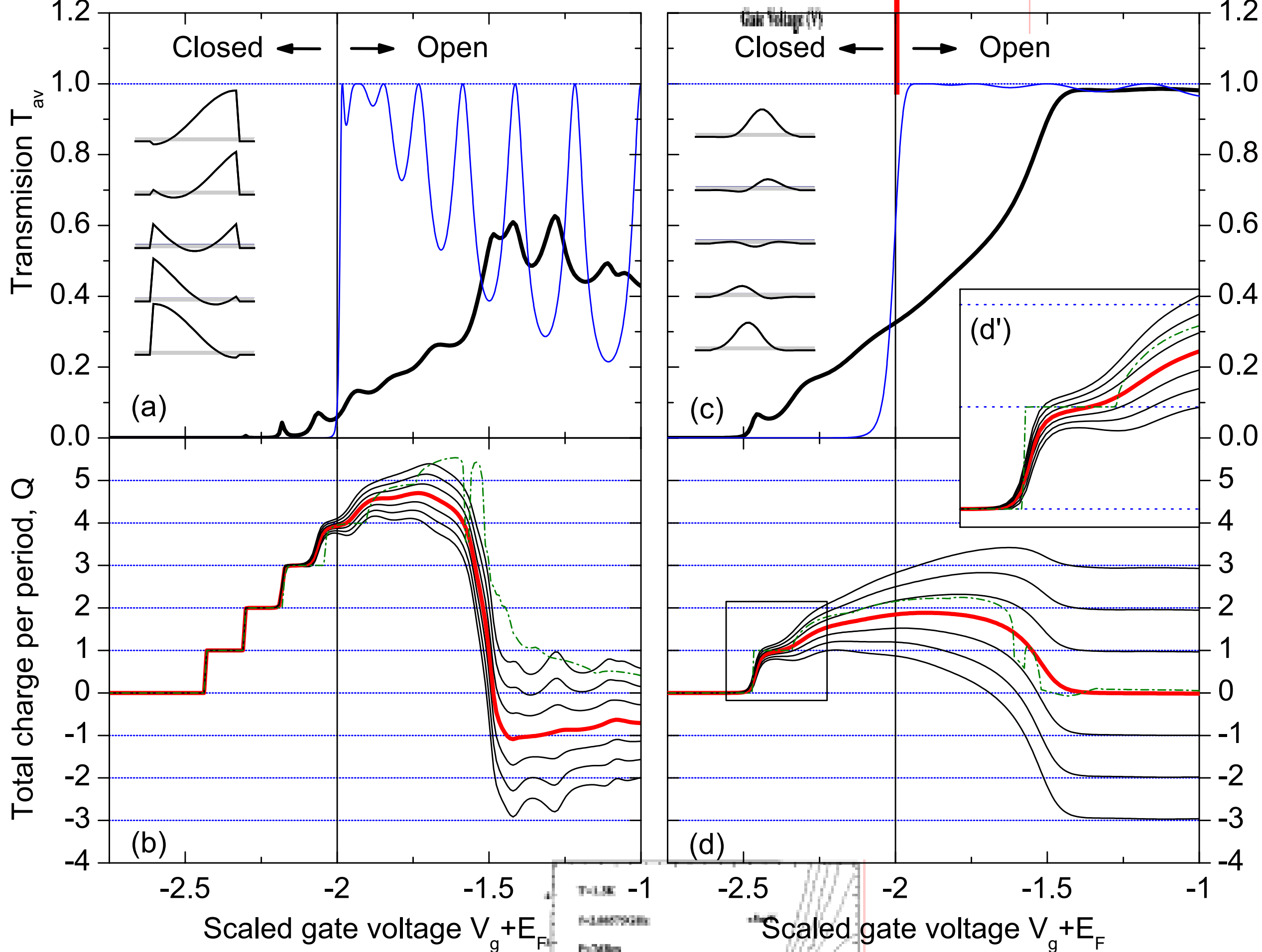
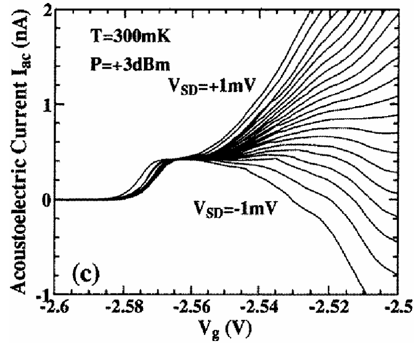


More details to discuss!





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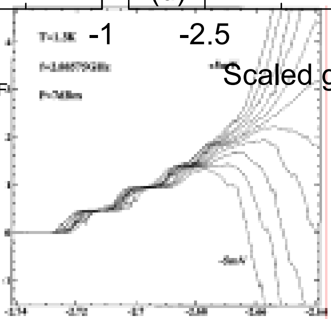
- Smearing of conductance pinch-off
- Combination of source-drain bias and pumping
- Role of gradual screening



Talyanskii *et al.*,
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Open questions

Interaction and correlation effects

For non-interacting electrons, we have a formalism to study the crossover from an open to a closed system. Can the resonance approximation be extended to include the **dynamic formation** of:

- Coulomb blockade?
- Kondo-type resonances?

The role of quantum interference

For kT >> Γ, the interference in the leads can be ignored and the resonance approximation is equivalent to a rate equation approach [4, cond-mat version].

- What is truly quantum in “adiabatic quantum pumping”?
- How to define a classical limit?
- Are there any kind of “intensity” versus “interference” terms?