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Azimuthal instability of radiation in gyrotrons with overmoded resonators

O. Dumbrajs[a)]
Institute of Solid State Physics, University of Latvia, Kengaraga strasse 8, LV-1063, Riga, Latvia and Helsinki University of Technology, Association Euroatom-Tekes, Finland

G. S. Nusinovich[b)]
Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742-3511

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Stability of efficient operation at one of the high-order modes is of great importance for the development of megawatt-level gyrotrons intended for plasma experiments in controlled fusion reactors. Typically such gyrotrons operate at modes with large azimuthal indices, which form a rather dense spectrum of eigenfrequencies. Therefore, instead of considering interaction of electrons with a large number of such modes it is more convenient to analyze the spatial-temporal evolution of an envelope formed by a superposition of these modes with the electrons. In all previous studies of stability of such envelopes it was assumed that some kind of azimuthal nonuniformity is present in the initial condition for the wave envelope. However, the physical reason for this nonuniformity, which is apparently the nonuniformity of the electron emission, was not analyzed. In the present paper, the relation between the emission nonuniformity and resulting nonuniformity of the wave envelope is established. Then, results of numerical simulations are given, which demonstrate various changes in the gyrotron dynamics caused by the azimuthal instability of the wave envelope. These results allow one to determine the maximum azimuthal index of the operating mode and show that this maximum index can depend on the degree of azimuthal nonuniformity of the electron emission. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1900603]

I. INTRODUCTION

An increasing scale of thermonuclear controlled reactors dictates continuous demands on increasing the operating frequency, power, and pulse duration of millimeter-wave gyrotrons intended for plasma start-up, electron cyclotron resonance heating and current drive, suppression of neoclassical tearing modes, and other applications (see, e.g., Refs. 1–3). In order to handle Ohmic losses of megawatt or even multi-megawatt level of microwave power in gyrotron resonators in continuous-wave (CW) regimes, the gyrotrons should operate at very high-order modes.[4–6] Such modes form a very dense spectrum of eigenfrequencies; therefore the problem of mode selection, competition, and startup scenario for such gyrotrons is of great importance.[7,8]

When a number of modes capable of simultaneous resonance interaction with gyrating electrons becomes very large, it makes sense instead of using a modal representation of the rf field to use a space-time description of it. This means representation of the rf field as a carrier azimuthally rotating wave multiplied by the wave envelope, which in the case of many modes in a resonator can be treated as a slowly variable function of time and azimuthal coordinate: $E = \mathrm{Re}\{f(t,\varphi)e^{i(\omega_0 t - m_0\varphi)}\}$. When diffractive quality factor (Q factor) greatly exceeds its minimum value, the axial structure of rf field in it can be considered in a so-called cold-cavity approximation that means that this structure is fully determined by the profile of resonator walls. The evolution of the wave envelope in such an approximation was first formulated in Ref. 9. In high-power gyrotrons, however, for minimizing Ohmic losses the diffractive Q factor is close to its minimum value, $Q_{\mathrm{dif,min}} \approx 4\pi(L/\lambda)^2$ (here L and λ are the resonator length and operating wavelength, respectively).[10] In such a case the axial structure of the rf field depends on the presence of an electron beam. Correspondingly, the wave envelope of a multimode radiation should be considered as a slowly varying function of time, azimuthal and axial coordinates, $f = f(t,\varphi,z)$. Equations describing this problem were first formulated and studied in Ref. 11. A more detailed analysis of this problem was later performed in Refs. 12 and 13. Results of these studies had allowed researchers to determine the maximum value of the normalized azimuthal period, $W = \pi\beta_{\perp 0}^2 m_0$, at which stationary generation of microwave oscillations is possible. For a gyrotron with a given orbital electron velocity normalized to the speed of light, $\beta_{\perp 0}$, this value $W_{\max}$ determines the maximum azimuthal index of the operating mode at which one should expect a stable, high-efficiency operation. At larger indices, the radiation becomes azimuthally unstable and efficiency drops significantly. This azimuthal instability causes quite chaotic temporal variations in the rf field amplitude.[11]

In all previous studies of the envelope evolution and corresponding azimuthal instability of gyrotron radiation, the following initial condition for the rf field envelope was used:

[a)]Electronic mail: dumbrajs@csc.fi
[b)]Electronic mail: gregoryn@umd.edu

$$f(\tau=0) = \left[0.1 + 0.01\sin\left(2\pi\frac{w}{W}\right)\right]\sin\left(\pi\frac{\varsigma}{\varsigma_{out}}\right). \quad (1)$$

Here $w=(\beta_{\perp 0}^2/2)m_0\varphi$ is the normalized azimuthal coordinate (m_0 is the azimuthal index of the carrier rotating wave), $\varsigma=(\beta_{\perp 0}^2/2\beta_{z0})(\omega_0 z/c)$ is the normalized axial coordinate, and ς_{out} is the correspondingly normalized exit coordinate. Certainly, the initial condition given by Eq. (1) is only one partial case of possible initial conditions, and it is not clear whether the nonstationary dynamics of overmoded gyrotrons depends on the choice of initial condition or not. Second, condition (1) itself does not explain the reason for appearance of this initial azimuthal nonuniformity in the rf field distribution. In view of recent theoretical[14–16] and experimental[17,18] studies of the effect of azimuthal nonuniformity of electron emission on gyrotron operation, it seems reasonable to assume that just this nonuniformity of electron emission causes the azimuthal nonuniformity of gyrotron radiation, at least, during nonstationary transition processes.

Present work is focused on the analysis of the effect of these two factors [viz. (a) the initial degree of nonuniformity of the rf envelope in initial conditions similar to Eq. (1) and (b) initial azimuthal nonuniformity of electron emission] on azimuthal instability of gyrotron operation. Such an analysis should allow researchers to better understand the origin of the azimuthal instability and conditions for its realization. The results obtained can also help to determine the maximum azimuthal index of stable operating mode in gyrotrons with a known degree of azimuthal nonuniformity of the emission.

The paper is organized as follows: Sec. II contains the formalism describing the evolution of rf field envelopes in gyrotrons with overmoded resonators. Section III is devoted to the analysis of correspondence between the emission nonuniformity and resulting nonuniformity of the rf field envelope in the framework of the small-signal theory. Section IV contains some results of numerical simulations. Finally, Sec. V contains the discussion of the results and the summary.

II. GENERAL FORMALISM

The self-consistent set of equations describing the nonstationary processes in gyrotrons with nonfixed axial and azimuthal structures of the rf field can be written as[11]

$$\frac{dp}{d\varsigma} + i(\Delta + |p|^2 - 1)p = if(\varsigma,w,\tau), \quad (2)$$

$$\frac{\partial^2 f}{\partial\varsigma^2} - i\frac{\partial f}{\partial\tau} - i\frac{\partial f}{\partial w} + \delta f = \frac{I}{2\pi}\int_0^{2\pi} p\,d\vartheta_0. \quad (3)$$

Here Eq. (2) is the standard Yulpatov equation for electron motion in gyrotrons: p is the complex transverse momentum of a gyrating electron normalized to its initial absolute value, $\Delta=2(\omega_0-\Omega_0)/\beta_{\perp 0}^2\omega_0$ is the normalized cyclotron resonance mismatch between the carrier frequency of the rf field and the initial electron cyclotron frequency Ω_0. In this paper only the fundamental cyclotron resonance between electrons and the rf field is considered; corresponding generalization for the case of arbitrary cyclotron harmonics is obvious. Equation (3) describes the temporal, axial, and azimuthal evolution of the wave envelope excited near cutoff in the process of interaction with electrons; here parameter $\delta=(8\beta_{z0}^2/\beta_{\perp 0}^4)\times[(\omega_0-\omega_{cut})/\omega_0]$ describes the deviation of the wave cutoff frequency in a waveguide of a variable radius with respect to the carrier frequency $\omega_{cut}(\varsigma)$ (below we assume the waveguide radius to be constant in the interaction region and, hence $\delta=0$), the normalized time is $\tau=(\beta_{\perp 0}^4/8\beta_{z0}^2)\omega_0 t$, and I is the normalized beam current parameter:

$$I = 16\frac{eI_b}{mc^3}\frac{\beta_{z0}}{\gamma_0\beta_{\perp 0}^6}\frac{J_{m_0\pm 1}^2(\omega_0 R_0/c)}{(\nu_0^2-m_0^2)J_{m_0}^2(\nu_0)}. \quad (4)$$

Here I_b is the beam current, e and m are the absolute value of electron charge and electron mass, respectively, ν_0 is the eigennumber for the carrier waveguide mode with the azimuthal index m_0, minus and plus signs in the order of Bessel function in the numerator correspond to the corotating and counterrotating waves with respect to the direction of electron gyration, R_0 is the radius of electron guiding centers in a thin, annular electron beam. The right-hand side of Eq. (3) contains averaging of the source term over initial gyrophases of electrons, ϑ_0, which in a nonmodulated electron beam at the entrance are uniformly distributed from 0 to 2π.

The boundary condition for electrons at the entrance is $p(\varsigma=0)=e^{i\vartheta_0}$. Equation (3) should be supplemented with the condition of azimuthal periodicity $f(w+W)=f(w)$, initial condition for the rf field distribution similar to Eq. (1), which will be discussed below in detail, and the boundary conditions for the rf field envelope at the entrance and the exit. At the entrance, where we assume the cutoff narrowing of the waveguide wall for preventing any leakage of the rf field into the electron gun region, we can assume zero field amplitude: $f(\varsigma=0)=0$. If we assume that the exit coordinate in a waveguide of a constant radius is determined by a sharp change in the external magnetic field, which leads to the interruption of cyclotron resonance interaction between the rf field and gyrating electrons, as it was done in Ref. 10, then at the exit the so-called reflectionless boundary condition for the outgoing radiation can be used (see also Ref. 19).

For the case of nonstationary operation near cutoff, this condition, as shown in Ref. 20, can be given as

$$f(\varsigma_{out},\tau) = -\frac{1}{\sqrt{\pi i}}\int_0^{\tau}\frac{1}{\sqrt{\tau-\tau'}}\left.\frac{\partial f(\varsigma,\tau')}{\partial\varsigma}\right|_{\varsigma_{out}}d\tau'. \quad (5)$$

This integro-differential equation is the result of the inverse Laplace transformation of the outgoing radiation condition for a single Fourier component of the nonstationary rf field, $f_\Omega=\int_0^\infty f(\tau')\exp(-i\Omega\tau')d\tau'$: $\partial f_\Omega/\partial\varsigma|_{\varsigma_{out}}=-i\gamma f_\Omega(\varsigma_{out})$; here γ is the dimensionless axial wave number whose normalization corresponds to normalization of ς (for more details see Ref. 20). In principle, one can also consider a more realistic configuration where a regular part of a waveguide, which plays a role of the interaction region, is joined in the output cross section with a slightly up-tapered output waveguide. Corresponding boundary condition for the case of nonstationary operation can be found in Ref. 21.

Finally, the self-consistent set of equations (2) and (3) should be augmented with expressions determining a so-called orbital efficiency of a single beamlet

$$\eta_{\perp,s} = 1 - \frac{1}{2\pi}\int_0^{2\pi} |p(\varsigma_{\text{out}})|^2 d\vartheta_0, \tag{6a}$$

which characterizes the extraction of the kinetic energy from electrons gyrating about the same guiding center and an orbital efficiency of a whole electron beam,

$$\eta_{\perp} = 1 - \frac{1}{W}\int_0^{W}\left[\frac{1}{2\pi}\int_0^{2\pi} |p(\varsigma_{\text{out}})|^2 d\vartheta_0\right] dw, \tag{6b}$$

which is the efficiency value averaged over all beamlets with different azimuthal coordinates of electron guiding centers.

Before closing this section let us note that such representation of the rf field as an azimuthally rotating envelope having certain carrier frequency and carrier azimuthal index is valid not only for conventional cylindrical but also for coaxial resonators. In the latter case, the normalized beam current parameter given above by Eq. (4) should be properly modified as is done in Ref. 22.

III. SMALL-SIGNAL THEORY: CORRESPONDENCE BETWEEN THE ELECTRON EMISSION AND rf FIELD ENVELOPE AZIMUTHAL NONUNIFORMITIES

As was stated in Sec. I, one of our goals was to analyze the effect of initial conditions on the azimuthal instability of gyrotron operation. Therefore, it was necessary to replace Eq. (1) by a more realistic initial condition. On the way to it, we have started from evaluating the initial level of spontaneous radiation from an electron beam consisting of an ensemble of uncorrelated charged particles. In this evaluation we followed Ref. 23, where such a procedure was described and it was suggested to estimate the level of a white noise as $N_*^{-1/2}$ where N_* is the number of particles passing through the cavity during a cavity decay time $\tau_d \sim Q/\omega$: $N_* = (I_b/e) \times (Q/\omega)$. For such typical gyrotron parameters as the beam current of 50 A, Q factor of 10^3, and frequency of 140 GHz, this number is close to 3.55×10^{11} that is close to the estimate 10^{12} given in Ref. 23. Thus, corresponding order of magnitude of the rf field amplitude in the initial condition should be on the order of 10^{-6}, but not 10^{-1} used before. However, when we started to perform simulations with very small values of the initial amplitude, it was found that the solutions are prone to some numerical instabilities, which do not allow us to distinguish the azimuthal instability having the physical nature from the numerical ones. Therefore, it was found reasonable to analyze the stage of the initial growth of the rf field amplitude from the white-noise level to the level of 0.1 with the use of the linear theory.

In the framework of the linear theory, the action of the rf field upon electrons should be treated as a small perturbation. Let us carry out this small-signal analysis assuming that the above mentioned cold-cavity approximation is valid and, hence, we can separate the axial dependence of the rf field on its temporal and azimuthal dependences, represent the rf field envelope as $f(\varsigma,\tau,\xi) = F(\tau,\xi)f'(\varsigma)$, and treat the amplitude F as a small parameter in the equation for electron motion (2). Correspondingly, the normalized electron transverse momentum p in this equation can be represented as $p_{(0)} + Fp_{(1)}$. In the zero-order approximation, Eq. (2) has a solution $p_{(0)} = \exp\{i(\vartheta_0 - \Delta\varsigma)\}$. Hence, in the source term in Eq. (3), this zero-order term, being averaged over initial gyrophases, yields zero. Thus, the representation of the rf field envelope given above and the fact that the zero-order term in the source term equals zero allow us to rewrite Eq. (3) in the following form:

$$\frac{\partial F}{\partial \tau} + \frac{\partial F}{\partial w} = iF\left\{I'\int_0^{\varsigma_{\text{out}}}\left[\frac{1}{2\pi}\int_0^{2\pi} p_{(1)} d\vartheta_0\right] f'^* d\varsigma + h^2\right\}. \tag{7}$$

To transform Eq. (3) into Eq. (7), Eq. (3) was multiplied with $f'^*(\varsigma)$ and integrated over the interaction length; correspondingly, the normalized beam current was renormalized as $I' = I/\int_0^{\varsigma_{\text{out}}}|f'(\varsigma)|^2 d\varsigma$. Also, it was taken into account that in the cold-cavity approximation, the function describing the axial structure of the rf field obeys the standard equation

$$\frac{d^2 f'}{d\varsigma^2} + h^2 f' = 0, \tag{8}$$

where the squared normalized axial wave number is equal to

$$h^2 = \frac{4\beta_{z0}^2}{\beta_{\perp 0}^4}\frac{k_z^2}{(\omega_0/c)^2} = \frac{4\beta_{z0}^2}{\beta_{\perp 0}^4}\left(\frac{\omega_{\text{cold}}^2}{\omega_{\text{cut}}^2} - 1\right). \tag{9}$$

Here $\omega_{\text{cold}} = \omega_s' + i(\omega_s'/2Q_s)$ is the cold-cavity frequency given with the account for microwave losses described by the cavity quality factor Q_s. Since we are dealing with high-Q cavities, Eq. (9) can be rewritten as

$$h^2 = h_0^2 + i\gamma, \tag{10}$$

where $h_0^2 = (8\beta_{z0}^2/\beta_{\perp 0}^4)[(\omega_s' - \omega_{\text{cut}})/\omega_0]$ and $\gamma = (8\beta_{z0}^2/\beta_{\perp 0}^4) \times (1/2Q_s)$ is the normalized decrement of the rf field in an empty cavity. For typical gyrotron parameters, beam voltage of 90 kV, orbital-to-axial velocity ratio of 1.3, and the Q factor of 800, this decrement is close to 1.7×10^{-2}.

Substituting Eq. (10) into Eq. (7) and introducing $\hat{w} = \gamma w$, $\hat{\tau} = \gamma\tau$, $\hat{I} = I'/\gamma$, and $\hat{h}_0^2 = h_0^2/\gamma$ (all hats will be omitted below) transforms Eq. (7) into the following equation:

$$\frac{\partial F}{\partial w} + \frac{\partial F}{\partial \tau} + F = iF\left\{I\int_0^{\varsigma_{\text{out}}}\left[\frac{1}{2\pi}\int_0^{2\pi} p_{(1)} d\vartheta_0\right] f'^* d\varsigma + h_0^2\right\}. \tag{11}$$

Our next step is to find the perturbations in the electron motion, i.e., to determine $p_{(1)}$. To do this we can use a method proposed in Ref. 24. According to Ref. 24, the first step is to introduce a new variable describing the electron normalized transverse momentum, $b = p\exp\{-i(\vartheta_0 - \Delta\varsigma)\}$. This variable, in the zero-order approximation, is simply $b_{(0)} = 1$. Correspondingly, the equation of electron motion (2) in the first-order approximation $(b = 1 + |F| b_{(1)})$ reduces to

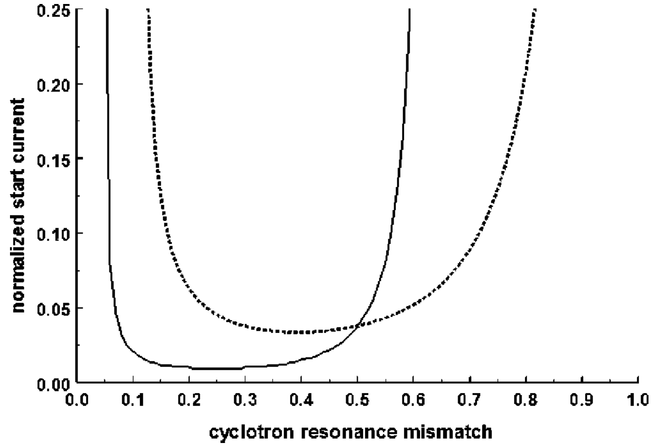

FIG. 1. Normalized start current as the function of the cyclotron resonance detuning Δ for two values of the normalized interaction length, $\varsigma_{out}=10$ (dotted line) and $\varsigma_{out}=15$ (solid line).

$$\frac{db_{(1)}}{d\varsigma} + i(b_{(1)} + b^*_{(1)}) = i\varphi, \tag{12}$$

where the function $\varphi(\varsigma)$ is equal to $f'(\varsigma)\exp\{-i(\vartheta_0-\Delta\varsigma-\psi)\}$; the rf envelope is represented here as $f(\varsigma,\xi,\tau)=|F|e^{i\psi}f'(\varsigma)$. As one can easily check, the solution of Eq. (12) can be given as

$$b_{(1)} = i\int_0^{\varsigma} \varphi(\varsigma')d\varsigma' + \int_0^{\varsigma}\left[\int_0^{\varsigma'}(\varphi-\varphi^*)d\varsigma''\right]d\varsigma'. \tag{13}$$

Now, we can come back from the variable $b_{(1)}$ to $p_{(1)}$, substitute solution given by Eq. (13) into Eq. (11), perform averaging over initial gyrophases, and rewrite Eq. (11) as

$$\frac{\partial F}{\partial w} + \frac{\partial F}{\partial \tau} + F = iF\{IS + h_0^2\}. \tag{14}$$

Here the source term S, which has the meaning of the electron beam susceptibility with respect to the rf perturbation, as follows from Eqs. (11) and (13), is equal to

$$S = \int_0^{\varsigma_{out}} f'^*(\varsigma)e^{-i\Delta\varsigma}\left\{ i\int_0^{\varsigma} f'(\varsigma')e^{i\Delta\varsigma'}d\varsigma' + \int_0^{\varsigma}\int_0^{\varsigma'} f'(\varsigma'')e^{i\Delta\varsigma''}d\varsigma''d\varsigma'\right\}d\varsigma, \tag{15}$$

i.e., it is a function of the cyclotron resonance mismatch Δ and the normalized length of the interaction space ς_{out}. As follows from Eq. (14), for exciting the rf oscillations from the noise level the imaginary part of the complex gain function, $S=S'+iS''$, should be negative, and the self-excitation condition for gyrotrons with an ideal electron beam can be given as

$$I \geqslant I_{start} = 1/(-S''). \tag{16}$$

The normalized start current as the function of the cyclotron resonance detuning is shown in Fig. 1 for two values of the normalized interaction length, $\varsigma_{out}=10$ and 15. These standard calculations were done for the axial structure of the rf field approximated by the sinusoidal dependence $f'(\varsigma)=\sin(\pi\varsigma/\varsigma_{out})$. Correspondingly, in calculations described below, the squared axial wave number h_0^2 in Eq. (14) was taken equal to $(\pi/\varsigma_{out})^2$.

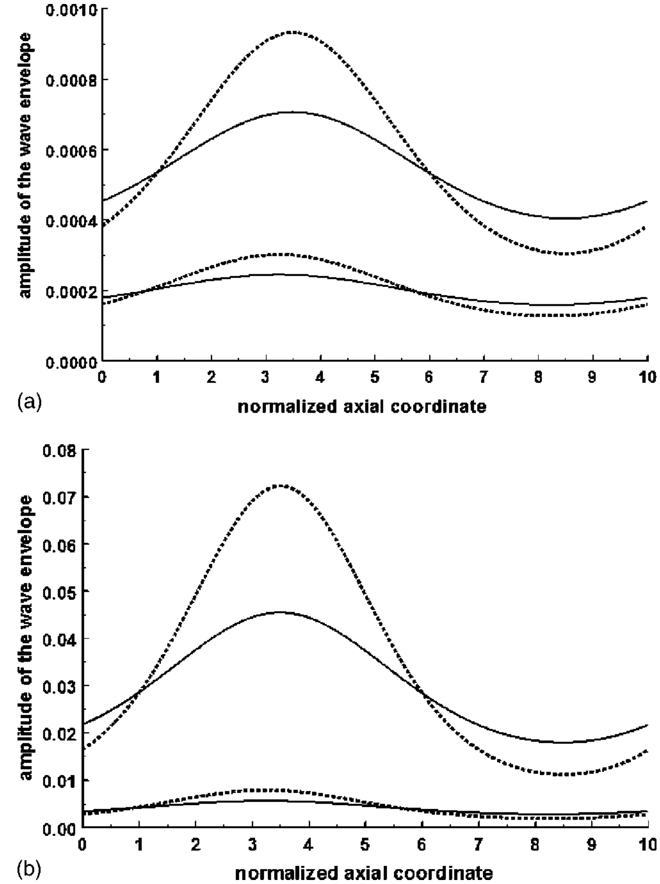

FIG. 2. Snapshots illustrating the temporal evolution of wave envelopes for $I_0/I_{0,start}=3$ (a) and $I_0/I_{0,start}=5$ (b). Solid and dotted curves correspond to the parameter α describing the azimuthal nonuniformity of electron emission equal to 0.05 and 0.1, respectively. Lower and upper pairs of curves correspond to the instants of normalized time equal to 1.5 and 2.0, respectively.

The normalized beam current parameter in the case under study is a function of the azimuthal coordinate $I=I(w)$. This azimuthal dependence can be either taken from some experimental data[17,18,25,26] or approximated by some analytical functions, which describe real distributions reasonably well. In our qualitative analysis we assumed that the normalized beam current is given as

$$I = I_0\left[1 + \alpha\sin\left(2\pi\frac{w}{W}\right)\right] \tag{17}$$

and studied the effect of the emission nonuniformity determined by the coefficient α on the rf envelope nonuniformity present in the solution of Eq. (14). The initial condition for the rf field amplitude in Eq. (14) was azimuthally uniform and equal to $F(0)=10^{-6}$. Results of calculations are shown in Fig. 2 for different ratios of the nominal beam current parameter to the start current, $I_0/I_{0,start}=3$ [Fig. 2(a)] and $I_0/I_{0,start}=5$ [Fig. 2(b)]. Here the snapshots show the azimuthal profile of wave envelopes for two different instants of the normalized time, $\tau=1.5$ and $\tau=2.0$; solid and dotted lines correspond to the emission nonuniformity parameter α

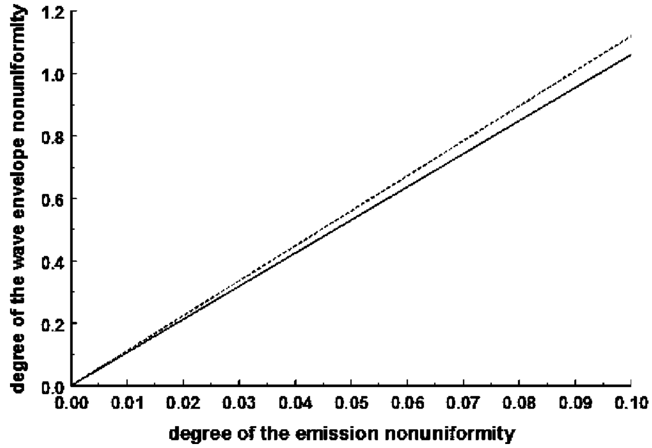

FIG. 3. Degree of the azimuthal nonuniformity of the wave envelope D_{env} as the function of the emission nonuniformity parameter α for $I_0/I_{0,start}=3$ (solid line) and $I_0/I_{0,start}=5$ (dashed line).

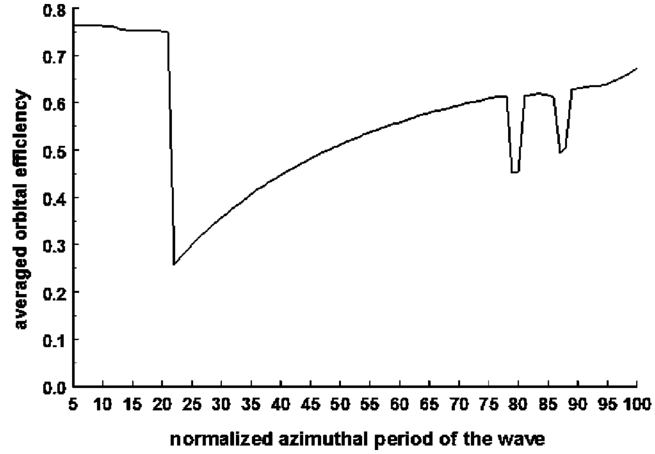

FIG. 4. Orbital efficiency of a whole electron beam as the function of the azimuthal period of the wave for the optimal gyrotron parameters and initial condition given by Eq. (1).

$=0.05$ and $\alpha=0.1$, respectively. As one can see, if at small values of α and small instants of time this dependence is close to the sinusoidal, at larger α's its shape differs from the sinusoidal distribution.

Resulting dependences of the degree of the azimuthal nonuniformity of the wave envelope on the degree of nonuniformity of electron emission α, are shown in Fig. 3 for $I_0/I_{0,start}=3$ and $I_0/I_{0,start}=5$. Here the degree of the wave envelope nonuniformity is defined as $D_{env}=(F_{max}-F_{ave})/F_{ave}$, where F_{ave} is the average value of the wave amplitude calculated for the case when the maximum value of the wave envelope, F_{max}, is close to 0.1. So, although the transition time from the initial noise level to 0.1 level of the wave envelope depends on the excess of the current over its threshold value, resulting degree of the wave nonuniformity as the function of the emission nonuniformity is practically the same: $D_{env}\simeq 10\alpha$. These results were obtained for $W=10$. Calculations done for $W=30$ and 50 resulted in very similar dependences.

IV. NONLINEAR THEORY

In this section we present some results of calculations of the set of nonlinear equations (2) and (3) and efficiencies determined by Eqs. (6a) and (6b). The initial condition for Eq. (3) is taken in the form similar to Eq. (1):

$$f(\tau=0)=\left[a+b\sin\left(2\pi\frac{w}{W}\right)\right]\sin\left(\pi\frac{\varsigma}{\varsigma_{out}}\right). \qquad (18)$$

All calculations were done for the normalized length of the interaction space close to its optimal value[10] $\varsigma_{out}=15$. We started from using in Eq. (18) the same values of a and b as those given in Eq. (1) and used in the previous papers.[11–13] Also normalized parameters, the cyclotron resonance mismatch Δ, and normalized beam current I_0 were chosen to yield the high efficiency (at least, at small enough W's):[11] $\Delta=\Delta_{opt}=0.6$, $I_0=I_{0,opt}=0.008$; at small W's, this set of parameters corresponds to $\eta_\perp=\eta_{\perp,max}=0.75$. Resulting dependence of the total orbital efficiency given by Eq. (6b) on the azimuthal period of the wave W is shown in Fig. 4. These data agree with those given in Ref. 11: at $W=20$ the operation is stable and highly efficient, while at $W=30$ the operation is much less efficient (about 0.35) and, as shown in Fig. 1(b) of Ref. 11, different beamlets interact with the rf field with different efficiency and this efficiency is a strongly irregular function of time. Results shown in Fig. 4 show, first, that the critical value of the azimuthal period of the wave, which was approximately evaluated in Ref. 11 as $W_{cr}\simeq 25$, in fact, is equal to 22. Second, in the range of W's between 22 and 75, the efficiency is a gradually increasing function of W, which increases from 0.26 to about 0.6. Although from comparing the efficiency values for $W=22$ and $W=75$ one can conclude that the latter case can offer higher efficiencies and, hence, is preferable; at present it would be premature to make such optimistic conclusions. Recall that we are considering an idealized model of the gyrotron, in which many factors important for operation of real devices (e.g., velocity spread) are ignored. Therefore for designing high-power gyrotrons operating in very high-order modes simulations for more realistic conditions should be performed.

Typical examples illustrating the temporal dependence of the beamlet efficiency and the total efficiency are shown in Fig. 5 where (a)–(c) correspond to W equal to 21, 22, and 44, respectively. It should be noted that at $W=22$ [Fig. 5(b)] for a rather long time (up to $\tau\approx 600$) the device exhibits a highly efficient operation (the same as for $W=21$), and only at larger times the instability results in the efficiency drop accompanied with strong oscillations of the beamlet efficiency, which, as shown in Ref. 11, correspond to quasichaotic oscillations of the rf field amplitude. Recall that here we present some results of simulations for the set of equations (2) and (3) where the time is not normalized to the wave decrement as in Sec. III. This value of time, $\tau\approx 600$, for such typical gyrotron parameters as the beam voltage of 90 kV and the orbital-to-axial electron velocity ratio of 1.3, which yields the normalized orbital velocity close to 0.417, corresponds approximately to 2600 wave periods. For typical Q factors in the range of 500–1000, this means several decay times of the rf field in the gyrotron cavity. Also note that for a given orbital velocity of electrons this critical value of the

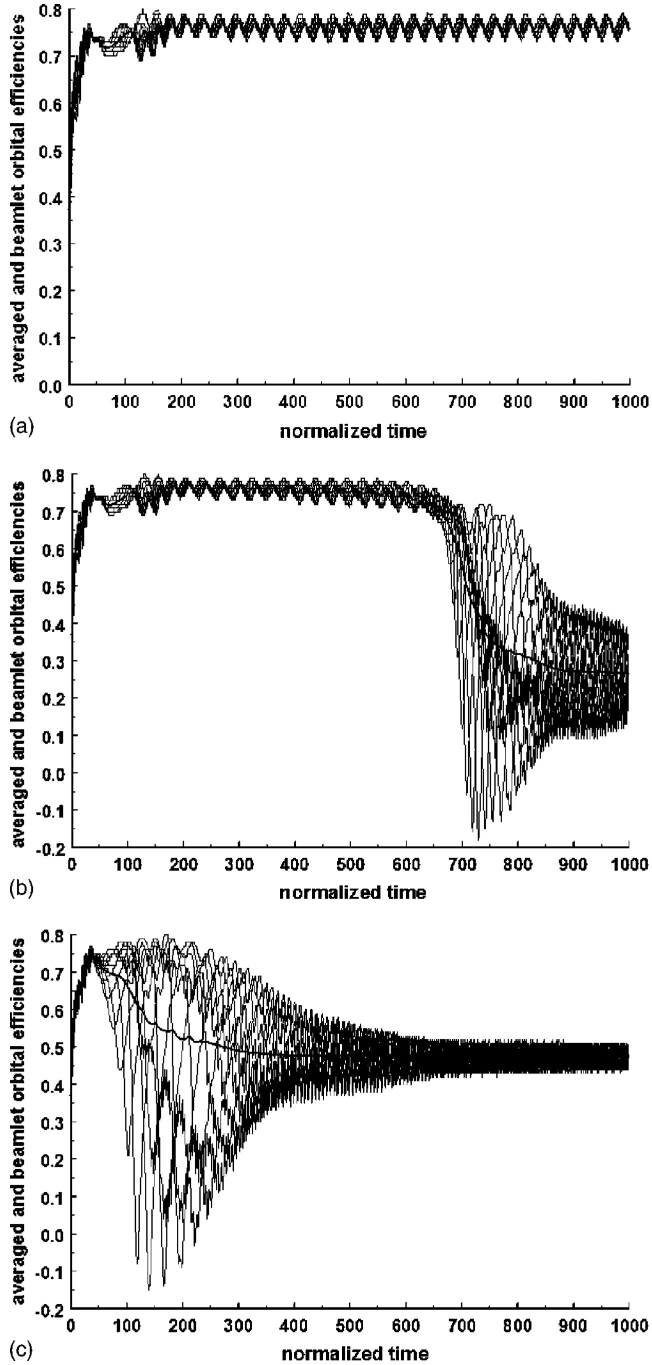

FIG. 5. Temporal evolution of the total efficiency (solid line) and efficiency of electron beamlets (thin line) for the same parameters as in Fig. 4 and several values of the azimuthal period of the wave: (a) $W=21$, (b) $W=22$, and (c) $W=44$.

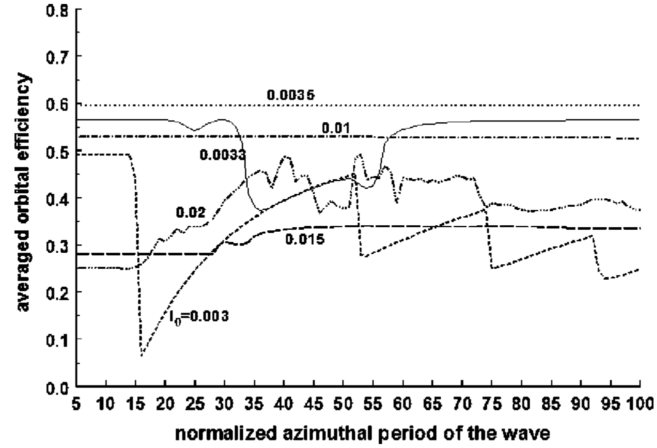

FIG. 6. Orbital efficiency of a whole electron beam as the function of the azimuthal period of the wave for different values of the beam current parameter. [Initial condition is given by Eq. (1), $\Delta=0.5$.]

azimuthal period, $W_{cr} \simeq 22$, corresponds to the azimuthal index close to 40, which should be considered as the maximum azimuthal index for a given set of gyrotron parameters. Third, at even larger values of this azimuthal period (larger than 75) there are two narrow regions of W's where the efficiency drops again, but not as strongly as at $W_{cr} \simeq 22$: from more than 0.6 to less than 0.5.

We also performed a number of simulations for a slightly different cyclotron resonance mismatch, $\Delta=0.5$, which also corresponds to rather highly efficient operation at proper values of the beam current parameter and the small enough azimuthal period of the rf field.[12,27] Results of simulations done for the same initial condition (1) and different values of the beam current parameter are shown in Fig. 6 as dependences of the total orbital efficiency given by Eq. (6b) on the azimuthal period of the wave W. At small beam current parameter value, $I_0=0.003$, which exceeds the start current by less than 50%, oscillations become unstable at rather small values of W: $W_{cr} \simeq 16$. Then, as the period increases, the efficiency gradually increases as well until the next critical value of the azimuthal period is reached where the efficiency drops again. In the range of W's up to 100, these critical values are equal to 53, 76, and 92. When the current increases from 0.003 to 0.0033, the gyrotron becomes able to sustain stable oscillations with 0.57 total orbital efficiency in a wide range of W's: up to 32 and then at $W>65$. (Note that in the range of W's between 22 and 32 there are some minor deviations in the efficiency.) In the intermediate range of W's between 32 and 65, the efficiency drops from 0.57 to about 0.4. An additional small increase in the current from 0.0033 to 0.0035 results in reaching the stable operation at all W's (up to 100) with rather high efficiency, $\eta_{\perp}=0.59$. This value of the normalized beam current parameter corresponds to the maximum efficiency for a given cyclotron resonance mismatch. Such a stable operation continues until the beam current parameter increases up to at least $I_0=0.01$; at this value of the beam current that is about three times larger than its optimal value the oscillations remain stable and the efficiency slightly decreases from 0.59 to 0.53. A further increase in the beam current parameter to $I_0=0.015$ results in a certain dependence of the gyrotron efficiency on the azimuthal period W: it is interesting to note that as the period increases from 28 to about 40 the total orbital efficiency of the beam increases from about 0.28 to about 0.35. Finally, the increase in the beam current to $I_0=0.02$ makes the dependence of the gyrotron efficiency on the azimuthal period even stronger: the operation is stable at W's up to 16 but the efficiency is there about 0.25 only, then as the period W

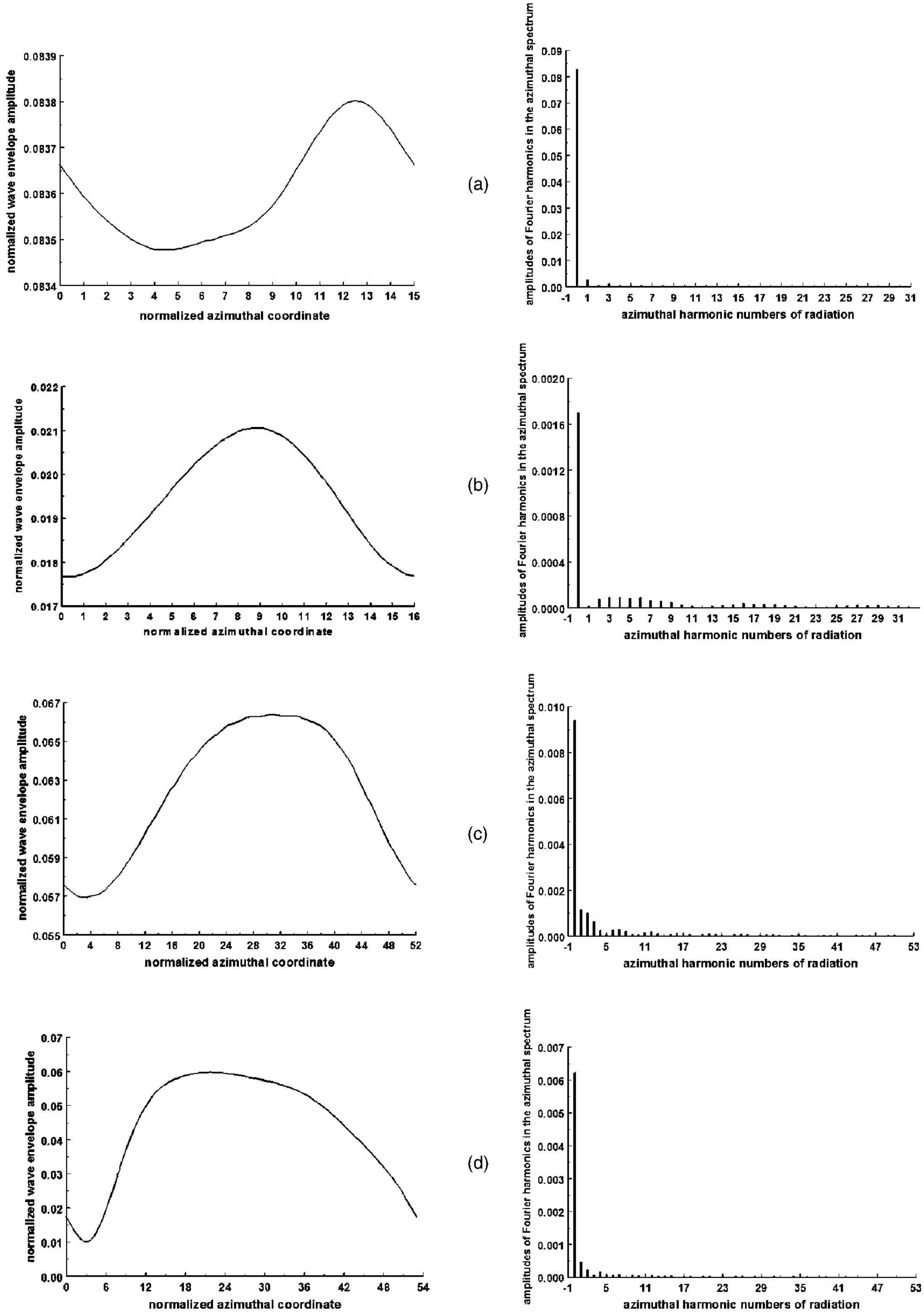

FIG. 7. Snapshots illustrating azimuthal structures of wave envelopes and corresponding azimuthal spectra for several values of the azimuthal period: (a) $W=15$, (b) $W=16$, (c) $W=52$, and (d) $W=53$.

increases the efficiency grows to the level up to about 0.5, but the dependence of the efficiency on W is rather irregular.

It is instructive to present some azimuthal distributions of the wave envelope and corresponding azimuthal spectra. Such data are shown in Fig. 7 for the normalized beam current parameter equal to 0.003 and several values of the azimuthal period of the wave envelope just below and above transitions from one steady state to another. (These transitions are shown in Fig. 6.) As one can see, these transitions correspond to significant changes in the azimuthal profile of the wave envelope and corresponding changes in the azimuthal spectrum. In line with results of Ref. 11, such enrich-

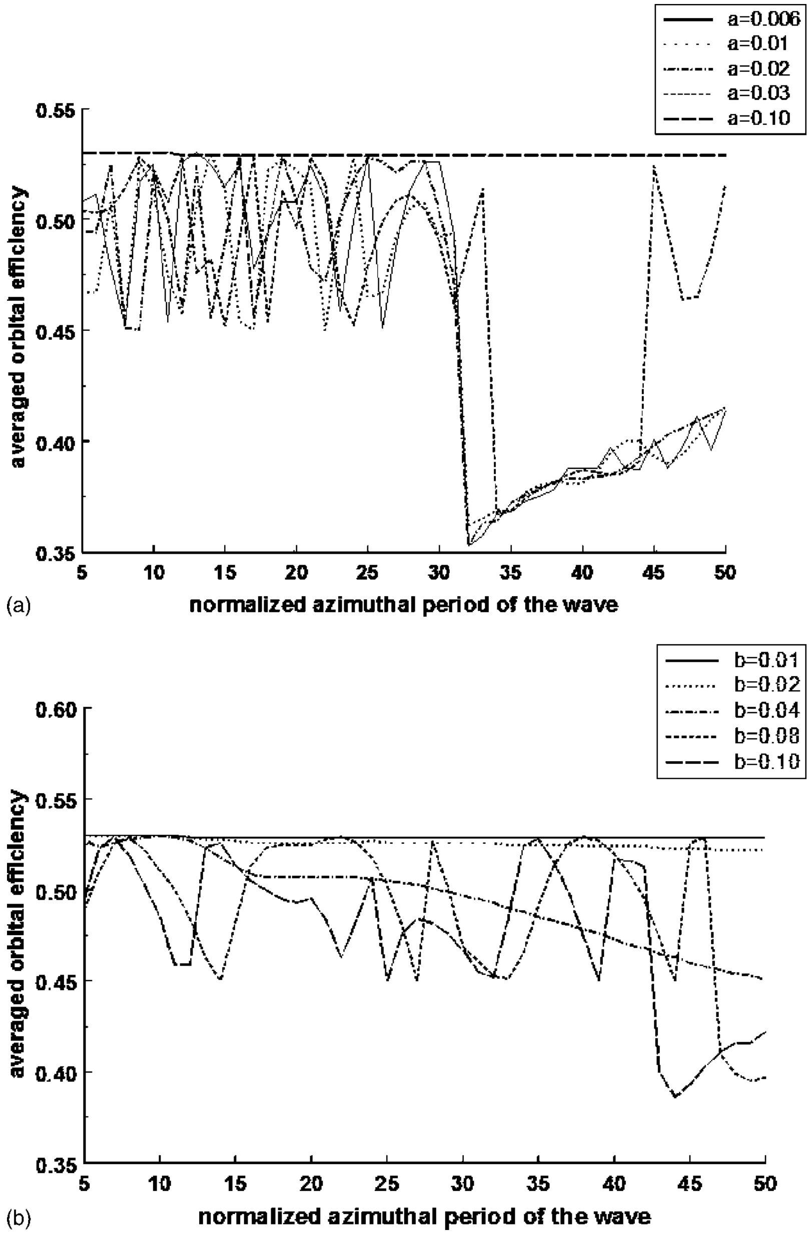

FIG. 8. Orbital efficiency averaged over the azimuthal distribution as the function of the azimuthal period of the wave for (a) given value of the initial rf nonuniformity of the rf field, $b=0.01$ and different values of a and (b) given mean value of the rf field, $a=0.1$, and different values of the parameter b characterizing the initial nonuniformity of the rf field.

ment of the azimuthal spectrum is also accompanied with more irregular temporal behavior of oscillations.

Next series of simulations were done for different values of the coefficients a and b characterizing the azimuthal nonuniformity of the rf field in the initial condition (18). Corresponding results are shown in Fig. 8 where Fig. 8(a) demonstrates the effect of variation in the amplitude of the rf envelope in the initial condition (18), a, for a given value of the initial azimuthal nonuniformity of the wave envelope, b, while Fig. 8(b) illustrates the effect of variation in b for a given value of a. As one can see, when the initial nonuniformity of the wave envelope is small enough the operation remains stable and efficient up to $W=50$. Thus, although this operation is less efficient (orbital efficiency is about 0.53) than that in the case of the optimal cyclotron resonance detuning shown in Fig. 4 (where at small W's this efficiency is about 0.76), it is more stable: it is stable even at $W=50$ while in the case optimal for efficiency, the efficiency dropped at $W=22$. The curve shown in Fig. 8(b) by a dotted line indicates that even when the b/a ratio is equal to 0.2, the efficiency remains practically unchanged at all W's up to $W=50$. At $b/a=0.4$ [this curve is shown by dash-dotted line in Fig. 8(b)], there is gradual degradation of the efficiency with increasing W, and, finally, when this b/a ratio is in the range of 0.8–1.0, one can observe drops in the efficiency with increasing W. In the range of W's smaller than 42, the efficiency drops from 0.53 to about 0.45, while at larger W's it drops to less than 0.4. Results shown in Fig. 8(a) demonstrate that the efficiency drop to less than 0.4 can be observed even starting from $W=32$–34, when the b/a ratio is larger than 0.3. In a certain sense, this result disagrees with the data shown for $b=0.04$ in Fig. 8(b) where there are no such drops at all W's under study. So not only the b/a ratio, but also absolute values of these coefficients can be important.

V. DISCUSSION AND SUMMARY

Results described above clearly demonstrate the importance of providing azimuthal uniformity of electron emission for stable gyrotron operation in high-order modes. Moreover, for the cathodes with a known degree of the emission nonuniformity, the formalism developed allows one to evaluate the maximum azimuthal index of the operating mode, which we identify as the index just before the first drop in the gyrotron efficiency. Then, taking into account the necessity to handle Ohmic losses in the gyrotron cavity operating in the CW regime dictates the choice of corresponding radial index. Hence, the choice of the operating mode for realizing the CW operation at a given power level and at a given frequency can be done with the use of these two factors. Of course, this choice also depends on parameters of electron beams such as the beam voltage and current and the orbital-to-axial electron velocity ratio. It should be emphasized that the conclusion made regarding the maximum azimuthal index is equally valid for both conventional cylindrical and coaxial resonators. Thus, the advantages of coaxial resonators in regard to the problem of mode selection are mostly determined by their ability to mitigate the voltage depression in the case of electron beams located far enough from the outer wall, which is the case of operating at modes with large radial indices. Note that just a strong azimuthal nonuniformity of the emission was possibly the reason for poor operation of the coaxial gyrotron described in Ref. 28. In this regard, it should be mentioned that the authors[28] had focused their interpretation of experimental results on the enlargement of the electron energy spread due to emission nonuniformity, but not on the azimuthal instability of radiation associated with this nonuniformity.

Results of our simulations also demonstrate various nonstationary processes in gyrotrons operating in very high-order modes and various sequences of events occurring with the increase in the azimuthal period of the wave envelope, i.e., with increasing the azimuthal index of the mode. It would be premature to claim that all physical effects leading to these phenomena are fully understood. However, the results presented above, at least, contain some new information and lead to new conclusions. In particular, important is the conclusion that one can realize more stable operation at high-order modes by slightly tuning the external magnetic field with respect to its value optimal for the efficiency.

In conclusion, let us note that some perturbations in the axial symmetry of the gyrotron interaction space can also be caused by misalignment of electron beam with respect to the resonator axis or, in the case of coaxial cavities, misalignment of an inner coax with respect to the outer wall.[29] Effects of this misalignment on the start currents and on the interaction between oppositely rotating waves were studied in Refs. 30 and 31, respectively. It should be noted that, although in Ref. 31 the treatment was focused on interaction between corotating and counterrotating waves having the same azimuthal index, the formalism developed there can be readily applied to the problem studied in the present paper. Indeed, a simple parallel shift of an inner coax with respect to the outer wall axis results in an admixture to a given mode a pair of modes differing in the azimuthal index by one from the operating mode. The presence of such modes can be interpreted as a perturbation in the initial condition (1).

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