Stochastization as a possible cause of fast reconnection in the frequently interrupted regime of neoclassical tearing modes

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The role of stochastization of magnetic field lines in fast reconnection phenomena occurring in magnetized fusion plasma is analyzed. A mapping technique is applied to trace the field lines of toroidally confined plasma where the perturbation parameter is expressed in terms of experimental perturbation amplitudes determined from the Axially Symmetric Divertor Experiment (ASDEX) Upgrade tokamak [S. Günter, C. Angioni, M. Apostoliceanu *et al.*, Nucl. Fusion **45**, S98 (2005)]. It is found that fast reconnection observed during amplitude drops of the neoclassical tearing mode instability in the frequently interrupted regime can be related to stochastization. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2136350]

An important goal of fusion research with magnetically confined plasmas is to maximize the achievable plasma pressure. In a tokamak, neoclassical tearing modes (NTMs), i.e., magnetic islands with poloidal (m) and toroidal (n) mode numbers driven unstable by the loss of bootstrap current inside the island, are of major concern as they are considered to be the most severe limitation to the maximum achievable plasma pressure in conventional tokamak scenarios. Such instabilities involve considerably large displacements of the plasma and can be described in the frame of magnetohydrodynamics (MHD). On ASDEX Upgrade a regime has been found[1] when the amplitude of the NTM after reaching a certain size suddenly drops to a much smaller value. After this the mode growth starts again. In this way the NTM amplitude never reaches its saturated value. This kind of neoclassical tearing modes was called frequently interrupted regime (FIR)-NTMs. In particular it has been observed that the amplitude of the $(m,n)=(3,2)$ NTM drops as an additional MHD instability [the $(m,n)=(4,3)$ mode] occurs. Generally the occurrence of the $(m+1,n+1)$ modes always coincides with the $(1,1)$ mode activity which is a necessary condition for the nonlinear coupling to the (m,n) NTM. The time in which these amplitude drops occur is very short (about 500 μs), much shorter than the resistive MHD reconnection rate (few 10 s of milliseconds in the ASDEX Upgrade). It has been suggested[2,3] that this experimental observation can be explained by stochastization of magnetic field lines when the island separatrix is destroyed. In this letter we present evidence in favor of this hypothesis.

Magnetic field lines can be regarded as trajectories of Hamiltonian systems. For the field line tracing two methods can be applied: (i) integration of the trajectory and (ii) mapping of the trajectory. The latter is a modern technique for the Hamiltonian system[4,5] and is implemented in our code. It is more than an order of magnitude faster than the integration. A properly chosen mapping procedure always conserves the main flux preserving property of the magnetic field, which is important for a correct reproduction of the long-term behavior of field lines in stochastic regions. In this formalism the equations for magnetic field lines take the Hamiltonian form

$$\frac{d\psi}{d\varphi} = -\frac{\partial H}{\partial \vartheta}, \quad \frac{d\vartheta}{d\varphi} = \frac{\partial H}{\partial \psi}, \tag{1}$$

where $\psi = r^2/2a^2$ is a toroidal magnetic flux, φ is a toroidal angle, ϑ is a poloidal angle, and a is a minor radius of the plasma (50 cm at the ASDEX Upgrade). The Hamiltonian H

$$H = H_0(\psi) + H_1(\psi, \vartheta, \varphi) \tag{2}$$

can be represented as a sum of the unperturbed flux

$$H_0(\psi) = \int \frac{d\psi}{q(\psi)} \tag{3}$$

and the perturbed part of the flux

TABLE I. Island width and corresponding perturbations.

(m,n)	W(cm)	W	$\Delta\psi$	r(cm)	r	ψ	ε
$(1,1)$	5	0.10	0.0050	15.8	0.316	0.050	2.4×10^{-4}
$(3,2)$	8	0.16	0.0128	29.6	0.592	0.175	1.5×10^{-3}
$(4,3)$	3	0.06	0.0018	25.8	0.516	0.133	3.2×10^{-4}

$$H_1(\psi,\vartheta,\varphi) = \sum_{m,n} H_{mn}(\psi)\cos(m\vartheta - n\varphi + \chi_{mn}). \tag{4}$$

Here $q(\psi)$ is the safety factor characterizing the winding of the magnetic field lines, $H_{mn}(\psi)$ is the perturbation Hamiltonian that corresponds to the perturbations of the modes (m,n) with the phases χ_{mn}.

For our purposes we have chosen the symmetric symplectic mapping derived in Ref. 5 on the basis of the Hamilton–Jacobi method [Eq. (30) in Ref. 5].

It is obvious that practical implementation of the mapping method requires knowledge of the safety factor and of the perturbation Hamiltonian. Determination of these quantities from the experiment is a challenging task, because of the large uncertainties in the measurements.

We have chosen the following parametrization for the safety factor:

$$q(\psi) = 0.8 + 4\psi, \tag{5}$$

which describes correctly the experimental position of the MHD modes at the ASDEX Upgrade.

The perturbation amplitude ε_{mn} in the Hamiltonian for each individual mode is defined as follows:[6]

$$\varepsilon_{mn} = B_{mn}/B_T, \tag{6}$$

where B_T is the toroidal magnetic field and B_{mn} is the magnetic perturbation due to the (m, n) mode. It can be roughly estimated from the width of the magnetic islands on the basis of the standard formula[7]

$$\Delta r_{mn} = 4\left[\frac{R_0}{m}\frac{B_{mn}(r)}{B_T}\left(\frac{dq^{-1}(r)}{dr}\right)^{-1}\right]^{1/2}\Bigg|_{r=r_{mn}} \tag{7}$$

and used in Eq. (6) for determination of ε_{mn}. In Eq. (7) R_0 is the major plasma radius. In normalized units $R_0 = 165$ cm/50 cm$=3.3$. In Table I we summarize the average parameters of the three islands observed in the experiment.

Approximation (7) becomes rather inaccurate for small perturbations due to large error bars for the island width (Δr_{mn}). Thus we try to estimate $B_{mn}(r)$ from magnetic measurements. As a first approximation, a representation of perturbation due to MHD mode can be written in the power law form[8,9]

$$H_{mn} = \varepsilon_{mn}\left(\frac{\psi}{\psi_{mn}}\right)^{m/2} \quad \text{for } \psi < \psi_{mn},$$

$$H_{mn} = \varepsilon_{mn}\left(\frac{\psi}{\psi_{mn}}\right)^{-m/2} \quad \text{for } \psi > \psi_{mn}, \tag{8}$$

where ψ_{mn} is the rational magnetic surface of the (m, n) mode. This representation corresponds to the so-called "step

current" approximation (SCA), which assumes that the perturbation current due to the mode has a step function structure and is strongly localized at the corresponding resonant surface with $q=m/n$. Such an assumption can be used as the zero order approximation for any type of MHD activities with resonant surfaces inside the plasma. SCA gives the correct asymptotic behavior ($\psi \sim r^{-m}$), but it cannot describe the perturbed flux close to the resonant surface (see Fig. 3 in Ref. 10). The MHD simulations, as well as the electron cyclotron emission (ECE) measurements, show a completely different behavior of the perturbation flux.[11] This difference can be attributed to the plasma influence which screens all the magnetic perturbations inside the plasma. This screening effect of the plasma is not taken into account in SCA which always underestimates the flux and hence has to be improved.

Parametrization (8) has two drawbacks: (i) incorrect shape of the perturbation flux close to the resonant surface, (ii) large errors of ε_{mn} in the case of small perturbations. These drawbacks can be eliminated by using the experimental information from the ASDEX Upgrade. We adopt the parametrization used in Ref. 11 for analyzing the structure of the (3,2), (4,3), and (5,4) modes:

$$H_{mn} = \rho_{mn}\alpha\left(\frac{\psi}{\psi_{mn}}\right)^{m/2}\left[1 - \beta\left(\frac{\psi}{\psi_{mn}}\right)^{1/2}\right] \quad \text{for } \psi < \psi_{mn},$$

$$H_{mn} = \rho_{mn}\frac{\alpha(1-\beta) - \gamma + \gamma\left(\dfrac{\psi}{\psi_{mn}}\right)^{1/2}}{\left(\dfrac{\psi}{\psi_{mn}}\right)^{(m+1)/2}} \quad \text{for } \psi > \psi_{mn}. \tag{9}$$

Here α, β, and γ are free parameters which fix the shape of the perturbation flux. The values of these parameters were determined in Ref. 11 from the analysis of ECE measurements of the (3,2) mode and provide us a correct form of the perturbations. The amplitudes of the perturbations B_{mn} can be directly deduced from the magnetic measurements which give us magnetic perturbations at the position of the magnetic probes located outside the plasma at $r=1.3$ ($\psi=0.845$). The normalization coefficients ρ_{mn} can be determined by demanding that the values of H_{mn} given by Eq. (9) coincide with the measured values at the position of the probes. In other words,

$$H_{mn}(\psi = 0.845) = H_{mn}^{\exp}(\psi = 0.845). \tag{10}$$

The same procedure can be employed for parametrization (8). In this case the parameter ε_{mn} is determined in the above described manner.

The resulting perturbations are shown in Fig. 1. In what follows we use the parametrization (8) only for the (1,1) mode. This mode is located at the plasma core where reconstruction of the perturbation flux from ECE is difficult.

As will be shown below, this mode has no influence on the stochastization but provides only a nonlinear coupling between other modes. Thus, the detailed structure of this mode is not so important as of the (3,2) and (4,3) modes.

It is interesting to compare the values $H_{11}^{\exp,\max}=8.6 \times 10^{-5}$, $H_{32}^{\exp,\max}=4.5\times10^{-4}$, $H_{43}^{\exp,\max}=2.2\times10^{-4}$ given by
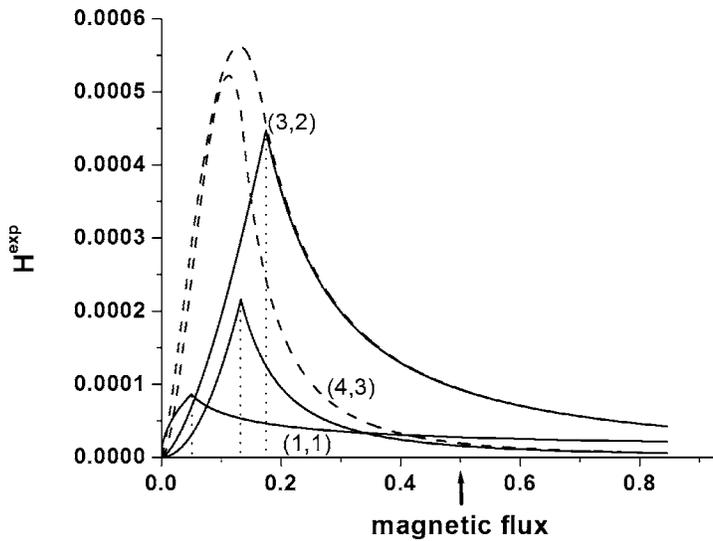
FIG. 1. Experimental perturbations for the (1,1), (3,2), and (4,3) modes as a function of ψ for the ASDEX Upgrade discharge No. 11681, $t=2.98$ s. Parametrization (8) (solid lines), parametrization (9) (dashed-dotted lines). For the (1,1) mode only the parametrization (8) is used. Here $\alpha=0.04$ and $\beta=0.87$ both for the (3,2) and the (4,3) mode, $\gamma=0.005$ for (3,2) and $\gamma=0.0005$ for (4,3), $H_{11}^{\exp}=2.1\times10^{-5}$, $H_{32}^{\exp}=4.2\times10^{-5}$, and $H_{43}^{\exp}=5.4\times10^{-6}$. Dotted vertical lines mark the positions of resonance surfaces. The plasma boundary is indicated by the arrow.

FIG. 3. The (1,1), (3,2), and (4,3) modes are used as perturbations.

parametrization (8) and the values $H_{32}^{\exp,\max}=5.6\times10^{-4}$, $H_{43}^{\exp,\max}=5.2\times10^{-4}$ given by parametrization (9) with the amplitude of the perturbations obtained from the island widths (Table I). SCA predicts the width of the island corresponding to the (3,2) mode of the order of 4.2 cm, while parametrization (9) gives about 5.1 cm, which is closer to the value 5.2 cm obtained in an independent ECE measurement.[2] For this reason in our calculations we use Eq. (9) instead of Eq. (8).

The results of the calculations performed with the (3,2) and (4,3) modes are shown in Fig. 2, and with inclusion of all three modes in Fig. 3, respectively. These Poincaré sections of field lines correspond to the time moment when all modes are locked together. The presence of the (4,3) mode
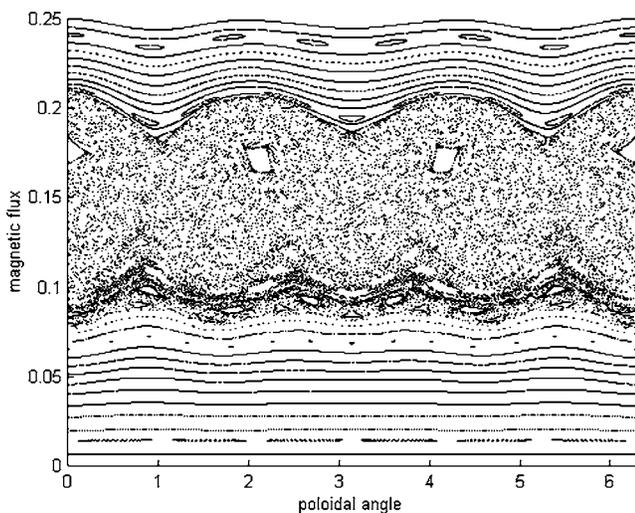

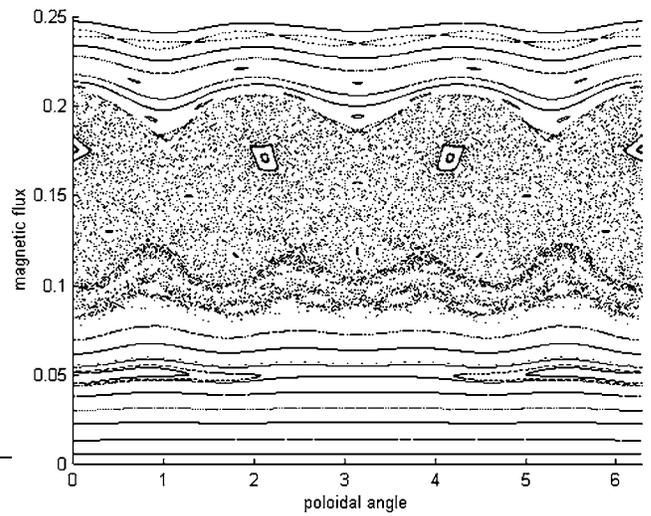FIG. 2. The (3,2) and (4,3) modes are used as perturbations.

immediately produces a large region of stochasticity (Fig. 2). This is due to the fact that, the perturbations of individual modes strongly overlap (Fig. 1). This overlapping is especially pronounced in the case of the (3,2) and (4,3) modes, because the distance between these modes is very small. On the one hand, the influence of the (1,1) mode on the stochastization itself is very weak (compare Fig. 2 with Fig. 3).

Its amplitude is rather small and it is located far from the (3,2) and (4,3) modes. Variations of the amplitude and the shape of this mode within the experimental errors do not change the overall picture. On the other hand, the presence of the (1,1) mode is needed for providing a nonlinear coupling between all the modes at the time point, when the modes are mutually locked and the stochastic region develops. As pointed out in Ref. 1, even a large (4,3) amplitude just before the occurrence of the mode coupling does not reduce the (3,2) amplitude. It drops only after the phase velocities of the (3,2) and (4,3) modes become equal.

We conclude that our calculations support the anticipation that the nonlinear interaction between the (3,2) and (4,3) modes leads to stochastization and that the presence of the (4,3) mode really is able to prevent the (3,2) mode from growing to its saturated island size. It was found in the ASDEX Upgrade that the FIR regime is not restricted to the interaction between the (3,2) and (4,3) modes. The same phenomenon has been observed in the case of the (4,3) and (5,4) modes. The results of the calculations for these modes are shown in Fig. 4. The physics of the interaction is exactly the same as in the case of the (3,2) and (4,3) modes. The only difference is the critical value of the amplitude for the (5,4) mode which is needed for creation of a stochastic region between the (4,3) and (5,4) modes. It was found that this critical value is by an order of magnitude smaller than the experimental value!

The reason for this is a very small distance between the (4,3) and (5,4) modes compared to the distance between the (3,2) and (4,3) modes. The resonances in this case completely overlap and even a tiny perturbation leads to stochastization. The stochastization does not occur, if the am-
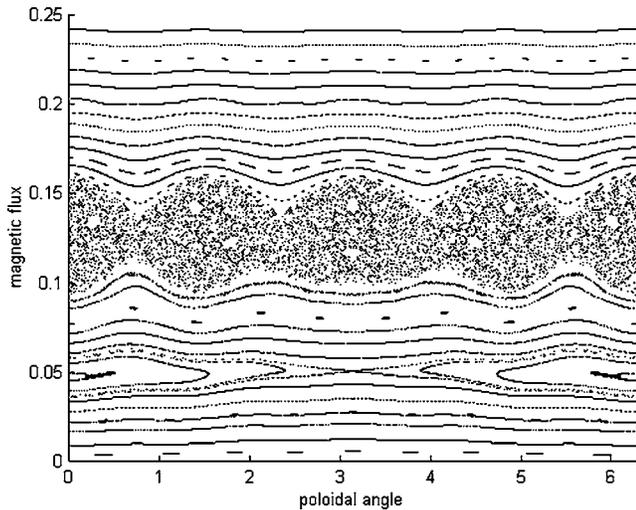
FIG. 4. ASDEX Upgrade discharge No. 11696, $t=2.98$ s. The (1,1), (4,3), and (5,4) modes are used as perturbations. Here $\alpha=0.04$, $\beta=0.87$ and $\gamma=0.005$ for both the (4,3) and the (5,4) mode, $H_{11}^{exp}=3.0\times10^{-5}$, $H_{43}^{exp}=5.0\times10^{-6}$, $H_{54}^{exp}=5.0\times10^{-7}$, $H_{11}^{exp,max}=1.2\times10^{-4}$, $H_{43}^{exp,max}=2.3\times10^{-4}$, $H_{43}^{exp,max}=8.2\times10^{-5}$.

plitude of the (5,4) mode is 20 times smaller than the experimental value. As was already pointed out in the previous example, to create a stochastic region one has to fulfill two conditions. First, the amplitude of the (5,4) mode must be larger than the threshold value, which indeed is the case. Second, all the modes have to be locked simultaneously. Without both of these conditions fulfilled, stochastization does not occur. Thus, for the (4,3) NTM, clearly the locking condition determines the onset.

In this Letter we have applied the mapping method for the field line tracing to investigate the role of stochastization in the FIR regime in the ASDEX Upgrade tokamak without going into details of the whole dynamics of this regime. Using the magnetic and ECE measurements we have reconstructed the safety factor profile and perturbations due to the MHD modes and implemented them into the Hamiltonian formalism of the field line tracing. As was pointed out, it is very difficult to determine accurately these quantities from the experiment. Moreover, the perturbation amplitudes differ by a factor 2 for different discharges and for different time periods of the same discharge. In spite of these difficulties our analysis of the FIR-NTM regime strongly suggests that the nonlinear interaction between the (3,2) and (4,3) modes leads to stochastization and that the presence of the (4,3)

mode really is able to prevent the (3,2) mode from growing to its saturated island size. Here the (1,1) mode, which is needed for a nonlinear coupling between the modes, has a negligible influence on stochastization itself. Similar conclusions refer to the interaction between the (4,3) and (5,4) modes. Variations of the perturbed fluxes and safety factor profile inside the error bars do not change this result.

The described investigations will be extended to the framework of a real ASDEX Upgrade geometry. On the theory side one should remember that magnetic reconnection is a universal process in which magnetic field lines embedded in a plasma break and reform, releasing large amounts of energy stored in stressed magnetic fields in the form of plasma flows and particle heating. For the MHD simulations, the observed stochastization can be represented by one parameter in Ohm's law, which is effectively the coefficient of electric current viscosity[12] that may lead to faster reconnection.[13] Here stochastization will be quantified by appropriate statistical methods and the corresponding heat conductivity equation will be solved.

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