Structure and dynamics of sawteeth crashes in ASDEX Upgrade

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The crash phase of the sawtooth in ASDEX Upgrade tokamak [Herrmann et al., Fusion Sci. Technol. 44(3), 569 (2003)] is investigated in detail in this paper by means of soft x-ray (SXR) and electron cyclotron emission (ECE) diagnostics. Analysis of precursor and postcursor (1,1) modes shows that the crash does not affect the position of the resonant surface $q=1$. Our experimental results suggest that sawtooth crash models should contain two ingredients to be consistent with experimental observations: (1) the (1,1) mode structure should survive the crash and (2) the flux changes should be small to preserve the position of the $q=1$ surface close to its original location. Detailed structure of the reconnection point was investigated with ECE imaging diagnostic. It is shown that reconnection starts locally. The expelled core is hot which is consistent with SXR tomography results. The observed results can be explained in the framework of a stochastic model. © 2010 American Institute of Physics. [doi:10.1063/1.3529363]

I. INTRODUCTION

In magnetically confined fusion plasmas, a variety of magnetohydrodynamic (MHD) instabilities can occur, driven by gradients of kinetic pressure or current density. The sawtooth oscillation is one of the fundamental instabilities in tokamaks, which is often observed but still has no definitive explanation for the crash process. This phenomenon is characterized by a repetitive and rapid crash of the central electron temperature.1 The Kadomtsev model,2 in which the $(m,n)=(1,1)$ island results in a new magnetic axis after the reconnection process, has provided a starting point for understanding the sawtooth, but not for an explanation of the phenomenon. Several experiments show that this model is in contradiction with experimental observations. For instance, it can explain neither the measured safety factors3–5 nor the existence of the (1,1) mode after the crash.6–7 It will be shown later that doubts about the validity of this model are found in ASDEX Upgrade experiments. As a result, a number of different theories was proposed to explain the crash dynamics and the short crash duration. In this paper we present results of observations of sawteeth crashes in ASDEX Upgrade.6 Investigation of sawtooth crashes in ASDEX Upgrade shows that in most cases magnetic reconnection is not complete.6 In fact, we do not have any cases with complete reconnection in hand. A postcursor mode is present in all analyzed cases, which means that all complete reconnection models are in contradiction with most experimental observations. It is also shown that some partial reconnection models are in contradiction with experimental observations in ASDEX Upgrade. In this situation, the stochastic model becomes a possible candidate for explanation of the sawtooth crash7,8 as will be discussed in what follows. In this paper we focus on the detailed dynamics and structure of the sawtooth crash.

II. COMPARISON OF THE $q=1$ POSITION BEFORE AND AFTER THE SAWTOOTH CRASH

Soft x-ray tomography is a standard way to visualize MHD processes in the hot plasma core. The soft x-ray diagnostics in ASDEX Upgrade consists of eight cameras with 208 lines of sight in total and acquisition frequency up to 2 MHz.9 In the following we analyze a typical sawtooth crash in ASDEX Upgrade. Initially, the plasma core is hot and can be seen as a rotated hot spot on the tomography [Fig. 1(a)]. The plasma core is cooled via exchange of temperature with the region outside $q=1$ during the crash and becomes cooler than the (1,1) island. The island is hotter after the crash, because it confines plasma from the initial region inside $q=1$ which is a bit hotter than the region outside of $q=1$. (Note that each figure has its own color scheme, to give the picture more contrast.)

During the crash phase an ideal (1,1) precursor mode converts into a (1,1) island. The ideal character of the precursor mode is well seen on electron cyclotron emission (ECE) time traces which show the same phase for all radial locations [Fig. 2(a)]. The postcursor island structure is visible on the tomography picture [Fig. 1(c)]. Tomographic reconstruction of the sawtooth crash also shows that the rotated precursor and postcursor modes are at the same radial location [see Figs. 1(a)–1(c)]. The position of the precursor and postcursor modes can also be identified directly from the soft x-ray (SXR) measurements without tomographic reconstruction and give the same result [Fig. 2(b)]. This result is shown to be typical. Several sawteeth in each of 30 randomly chosen discharges were analyzed for statistical purposes.
As an example, three different cases of incomplete sawtooth reconnection are shown in Fig. 3 where the fast Fourier transform (FFT) amplitude of the line integrated soft x-ray signals is plotted versus the soft x-ray line angle. The examined cases have different plasma parameters and different heating which result in different frequencies of the (1,1) mode.

The sawtooth precursor and sawtooth postcursor correspond to the same position of the \( q=1 \) surface and the same \( m=1 \) structure (one minimum in the plasma center). Moreover, in spite of a search, no cases with a pronounced reduction of \( q=1 \) radius were found. The accuracy of the measurements of the \( q=1 \) position with SXR cameras is about 2–4 cm which is about 10% of the diameter of \( q=1 \) resonant surface (30–40 cm in our cases). Direct analysis of SXR signals, as was done in Figs. 2(b) and 3, has the advantage that it is not affected by any regularization assumption which is necessary for tomographic inversion. Actually, determination of the (1,1) mode position is the best and most precise indication of the resonant surface position.

To understand the consequence of the result it is convenient to start from Kadomtsev’s model of the sawtooth crash. In this model, poloidal flux is annihilated during the reconnection process (reconnection in the strong toroidal field). Magnetic surfaces of equal helical flux reconnect such that toroidal flux is conserved. Based on these rules it is possible to determine the final safety factor profile after the crash (see Fig. 4). The result of the Kadomtsev process is complete reconnection in which the O-point of the (1,1) island becomes the new plasma center. Thus, the position of \( q=1 \) in our experiments is in clear contradiction with the Kadomtsev model which suggests \( q=1 \) after the crash only at the magnetic axis (see Fig. 4).

The subsequent relaxation of the current profile moves the \( q=1 \) surface slowly to its original position in this model.
The partial reconnection models are generally based on Kadomtsev’s model and assume that reconnection is stopped at a particular radius, \( r_{\text{inner}} \), which is a new free parameter in these models. As an example, we discuss here the Porcelli model.\(^{11}\) In this model reconnection stops at the inner radius \( r_{\text{inner}} \). Thus, inside this radius the safety factor is frozen. At the end of the reconnection process, a ring with \( q = 1 \) and constant helical flux is formed between the inner and the outer radius. Thus, as a result of the crash, no helical structures are present and the hot core is confined. This variant is in clear contradiction with our experimental observations because it contains no postcursor mode and has no heat flow from the core region as seen in experimental observations. Because it contains no postcursor, Kadomtsev’s model and assume that reconnection is stopped at a particular radius, \( r_{\text{inner}} \), which is a new free parameter in these models. As an example, we discuss here the Porcelli model.\(^{11}\) In this model reconnection stops at the inner radius \( r_{\text{inner}} \). Thus, inside this radius the safety factor is frozen. At the end of the reconnection process, a ring with \( q = 1 \) and constant helical flux is formed between the inner and the outer radius. Thus, as a result of the crash, no helical structures are present and the hot core is confined. This variant is in clear contradiction with our experimental observations because it contains no postcursor mode and has no heat flow from the core region as seen in Fig. 1. In a later paper,\(^{12}\) Porcelli formulates a Hamiltonian ansatz from Kadomtsev’s model and at the same time provides the mode at the right position (around the original \( q = 1 \) surface). But the flux surfaces inside the inner core are still not affected during the crash. Thus, heat flow from the core is not possible, which is in contradiction with experimental observations.

In summary, our findings provide a set of restrictions on a possible sawtooth crash model:

1. Constant position of the resonant surface restricts strong modification of the poloidal flux.
2. The mode with \( (1,1) \) helicity should survive the crash phase.
3. Heat comes out from central core region through the X-point of the \( (1,1) \) mode.

It should be noted that partial reconnection models allow one to make good predictions for the sawtooth period in transport calculations.\(^{13}\) The reason for this good agreement is that these calculations are sensitive to changes of global quantities, which are defined by, for example, \( r_{\text{inner}} \), but are not sensitive to the exact mechanism of the crash and cannot be used to verify/falsify crash models.

III. CHANGES OF THE SAFETY FACTOR PROFILE IN A STOCHASTIC REGION

It was shown by means of a mapping technique that amplitudes of the primary \( (1,1) \) mode together with its harmonics are sufficient to stochastize the region if the central \( q \) is smaller than 0.85–0.9.\(^{7}\) This is in good agreement with measurements of the central safety factor profile (Refs. 3–5) and allows one to explain the existence of the mode after the sawtooth collapse. In the following, we investigate the local behavior of the safety factor for the same case as in Ref. 7 (see Fig. 9 in the Appendix). The safety factor profiles in the figures [Figs. 9(a)–9(c)] are equilibrium profiles. In equilibrium, by definition the safety factor value is a constant on a particular flux surface, i.e., each field line has the same \( q \)-value. The situation is different in a stochastic region. The flux surfaces no longer exist and two neighboring field lines would be shifted to completely different positions on a Poincare plot after several rotations of the field line. In spite of this difficulty it is still possible to define an average safety factor value for each field line. The standard definition of the safety factor is the ratio of the poloidal and toroidal paths of the magnetic field line: \( q = \Delta \phi / 2 \pi \), where \( \Delta \phi \) is the variation of toroidal angle after one full rotation in the poloidal plane. This definition reflects the topological property of the magnetic field line (the strength of winding of the line) and does not require existence of flux surfaces. Thus, the same definition can also be used in a stochastic region. The basic difference to the ordinary case is the fact that the \( q \)-value is no longer a flux surface constant but a field line constant. This allows one to define the average safety factor of a magnetic field line after many rotations around the torus as follows:

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**FIG. 3.** (Color online) FFT amplitude of the \( (1,1) \) mode depending on the line of sight for three cases of incomplete sawtooth crash (no. 23923, \( t_{\text{before}} = 2.167 \, 270 \, 4 \)–\( 2.169 \, 139 \, 4 \) s, \( t_{\text{after}} = 2.172 \, 420 \, 1 \)–\( 2.173 \, 803 \, 5 \) s; no. 24007, \( t_{\text{before}} = 2.966 \, 701 \, 3 \)–\( 2.969 \, 268 \, 4 \) s, \( t_{\text{after}} = 2.983 \, 278 \, 1 \)–\( 2.991 \, 885 \, 8 \) s; no. 23922, \( t_{\text{before}} = 2.611 \, 692 \, 6 \)–\( 2.612 \, 446 \, 4 \) s, \( t_{\text{after}} = 2.613 \, 023 \, 7 \)–\( 2.613 \, 635 \, 0 \) s). The frequencies of the \( (1,1) \) mode are different due to different heating power. One can see that the precursor \( (1,1) \) and postcursor \( (1,1) \) modes give the same position for the \( q = 1 \) surface. In some cases, even a small increase of \( q = 1 \) radius can be seen after the crash (no. 23922). No cases with reduction of the \( q = 1 \) radius after the crash were observed. One global minimum in the plasma center gives clear \( m = 1 \) structure of the precursor and postcursor.
where $N$ is the number of toroidal rotations of the magnetic field lines and $\theta_i$ is a poloidal position of the field line at $i$th iteration. This is just a reformulated definition of the safety factor averaged over a large number of toroidal rotations. The field line follows a helical path around the torus even in iteration. This is just a reformulated definition of the safety factor profile.

One can see that field lines in the island have safety factor values equal to unity which is an expected result. (The topological helicity of the island is $(m,n) = (1,1)$ which is exactly unity for safety factor.) The more surprising result is the value of safety factor in the stochastic region. In spite of the strong stochasticity and overlap of the regions with different safety factor values, the general features of the safety factor profile remain the same. The safety factor is smaller in the core and larger close to the island. This can be seen more clearly in Fig. 6(a), where the result is presented as a contour plot.

This is an averaged contour plot for the same case as in Fig. 5. The changes of the averaged safety factor values are shown across the O-point of the island (line A) and across the X-point (line B) in Fig. 6(b). The initial equilibrium profile is also shown (solid line). It is possible to introduce additional averaging along the poloidal angle and compare resulting profiles with the initial equilibrium. Here even in case of large stochasticity the average values of safety factor remain almost unchanged in the presence of perturbations (the central value has slightly increased from 0.7 to 0.8).

Thus, strong field line mixing has a small effect on the field line helicity. This example demonstrates that the stochastic model can resolve one of the main problems of the sawtooth crash. It provides simultaneously:

- Strong and fast equalization of temperature as shown for the experimental perturbations. This is a natural and general feature of stochastic regions in the plasma, due to high parallel heat conductivity along the magnetic field.
- Small changes in the safety factor profile (as it is shown here for the most stochastic case). This agrees with observations which suggested small changes of the safety factor value in the core after the crash. This also allows one to make much faster profile relaxation to the initial stage in the transport calculations which suffer from large changes of the safety factor profiles and require partial mixing to fit the experimental results.

After the crash, the (1,1) structure decays and vanishes after 5–20 rotations around the torus because there is no further drive. For less stochastic case [see Fig. 9(a) in the Appendix] the average value changes less because perturbations are very small as shown in Fig. 7.
IV. RECONNECTION REGION

Any changes of the field line topology require reconnection of the magnetic field lines. Thus, reconnection is necessary for any crash model because this is the only way to produce a (1,1) island structure after the crash. The main difference between the models is amount of reconnection and subsequent changes of safety factor profile. Kadomtsev’s model requires a much larger amount of reconnected flux compared to the stochastic case and much larger changes in safety factor profile as discussed in Secs. II and III.

The reconnection region is located in the X-point of the (1,1) mode for the sawtooth case. This region is rather local and cannot be resolved by SXR tomography [see Fig. 1(b)]. At the same time, the newly available ECE imaging (ECEI) diagnostics resolves the sawtooth crash locally if the crash is in the region of ECE measurements. First observations from ECEI confirm results from TEXTOR (Ref. 16) that position of the sawtooth crash has no preferred location in the poloidal plane and reconnection can happen at any poloidal position. An example of low field side reconnection from ECEI is shown in Fig. 8. This is the same crash as in soft x-ray tomography picture (Fig. 1).

V. CONCLUSIONS

Typical sawteeth crashes in ASDEX Upgrade were investigated in detail using all available measurements which could clarify the structure and dynamics of the crash phase. It was confirmed that most sawteeth are incomplete. The precursor and postcursor modes were used to identify the position of the resonant surface \( q = 1 \) just before and directly after the crash event. It was shown that the mode position remains the same and no indication of the reduction of \( q = 1 \) radius was found. Detailed structure of the reconnection point was investigated with ECE imaging diagnostic. The result allows one to conclude that reconnection starts locally.

Our experimental results suggest that any experimentally consistent sawtooth crash model should contain three ingredients:

1. The (1,1) structure should survive the crash.
2. The flux changes should be small which will preserve the position of the \( q = 1 \) surface.
3. Heat comes out from central core region through X-point of the (1,1) mode.

It is shown that these conditions can be fulfilled in a stochastic model of the sawtooth crash. Stochasticity provides fast equilibration of the temperature profile simultaneously with small changes in safety factor profile. At the same time, the model preserves the island at its original position. The values of the safety factor profile at the plasma axis were measured in some cases with MHD spectroscopy techniques using the toroidal alfvén eigenmodes (TAEs).
mode frequencies. The resulting values were found to be less than unity\textsuperscript{17,18} which agrees with other measurements\textsuperscript{3–5} and with the stochastic model assumption.

It should be noted that special cases of sawteeth are seldom observed in ASDEX Upgrade (for example, compound sawteeth, inverse sawteeth, etc.).\textsuperscript{19} These crashes were not investigated in this paper. We have focused on most common variants of the sawteeth.

APPENDIX: RECONSTRUCTION OF THE MAGNETIC FIELD LINES STRUCTURE

The sawtooth phenomenon was analyzed by means of Hamiltonian mapping in Ref. 7. The applied method uses experimental mode perturbations and different safety factor profiles to trace the field lines of the magnetic field. In the

FIG. 8. (Color online) ECE images of the sawtooth crash are shown for the same crash as in Fig. 1. These measurements support the clockwise rotation of the mode which is seen by SXR tomography. One can see start of the heat outflow through X-point.

FIG. 9. (Color online) Poincare plots for the same perturbations but different safety factor profiles. Note that stochastization strongly depends on the existence of the low-order rational surfaces which are marked on safety factor curves: (a) central $q$-value is 0.7, (b) central $q$-value is 0.85, and (c) central $q$-value is 0.9.
formalism the equations for magnetic field lines take the Hamiltonian form
\[
\frac{d\psi}{d\varphi} = -\frac{\partial H}{\partial \vartheta}, \quad \frac{d\vartheta}{d\varphi} = \frac{\partial H}{\partial \psi},
\]
where \(\psi = r^2/2a^2\) is a toroidal magnetic flux canonically conjugated to the poloidal angle \(\vartheta\), \(\varphi\) is a toroidal angle, and \(a\) is a minor radius of the plasma (50 cm at ASDEX Upgrade). The Hamiltonian \(H\)
\[
H = H_0(\psi) + H_1(\psi, \vartheta, \varphi)
\]
can be represented as a sum of the unperturbed flux
\[
H_0(\psi) = \int \frac{d\psi}{q(\psi)}
\]
and the perturbed part of the flux
\[
H_1(\psi, \vartheta, \varphi) = \sum_{m,n} H_{mn}(\psi) \cos(m \vartheta - n \varphi + \chi_{mn}).
\]

Here \(q(\psi)\) is the safety factor characterizing the winding of the magnetic field lines, \(H_{mn}(\psi)\) is the perturbation Hamiltonian which corresponds to the perturbations of the modes \((m,n)\) with the phases \(\chi_{mn}\). It is obvious that practical implementation of the mapping method requires knowledge of the safety factor and of the perturbation Hamiltonian. The perturbation Hamiltonian was reconstructed based on soft x-ray and electron cyclotron emission measurements. Accurate determination of the central \(q\)-profile is not possible in ASDEX Upgrade to the degree needed here and its influence was investigated by changing the safety factor profile. The result is shown in Fig. 9.


10V. Igochine, A. Gude, and M. Maraschek, IPP Report No. 1/338, 2010; see http://edoc.mpg.de/display.epl?mode=doc&id=476537


