AIP Physics of Plasmas

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Citation: Phys. Plasmas **19**, 103112 (2012); doi: 10.1063/1.4764073
View online: http://dx.doi.org/10.1063/1.4764073
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v19/i10
Published by the American Institute of Physics.

PHYSICS OF PLASMAS 19, 103112 (2012)

On optimization of sub-THz gyrotron parameters

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(Received 20 August 2012; accepted 10 October 2012; published online 25 October 2012)

The theory is developed describing how the optimization of gyrotron parameters should be done taking into account two effects deteriorating the gyrotron efficiency: the spread in electron velocities and the spread in the guiding center radii. The paper starts from qualitative analysis of the problem. This simplified theory is used for making some estimates for a specific gyrotron design. The same design is then studied by using more accurate numerical methods. Results of the latter treatment agree with former qualitative predictions. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4764073]

I. INTRODUCTION

At present, there is a strong interest in the development of sub-THz gyrotrons for various applications. Among such applications one can mention: the dynamic nuclear polarization enhancement of NMR spectroscopy,[1–4] which requires relatively low power (several Watts is typically sufficient), stable, and continuous-wave operation; direct measurement of the hyperfine transition of the ground state positronium,[5] which requires about 10 kW power; active plasma diagnostics;[6] and possible remote detection of concealed radioactive materials,[7] which requires hundred kW (or above) short-pulse power. As always, it is desirable to maximize the efficiency of the gyrotron operation that, in the case of sub-THz frequencies, has certain specific features[8,9] because the ohmic losses of THz power in the cavity walls become comparable to the power of outgoing radiation.

In recent years, a number of designs of magnetron electron guns for sub-THz gyrotron operation have been done with the goal to choose a cathode slant angle providing the minimum electron velocity spread.[10] It was shown that the minimum spread can be realized in electron guns forming laminar electron beams. Different types of formed electron beams are illustrated by Fig. 2.3 in Ref. 11. The radial width of such laminar beams is, however, larger than the radial width of the beams formed by the guns with smaller slant angles.

As known, the radial thickness of electron beams may result in the different strength of the beam coupling to the wave for electrons having different radial coordinates of electron guiding centers. This, in turn, may cause a certain deterioration of the gyrotron efficiency.[12]

The goal of the present study is to address this controversy between various methods of optimization of gyrotron parameters and to formulate some criteria which would help gyrotron designers to maximize the efficiency of sub-THz gyrotrons. The paper is organized as follows. In Sec. II we present a formulation which allows one to carry out a qualitative analysis of the problem. Section III describes the results based on this formulation. Section IV contains results of numerical simulations of the efficiency for one specific gyrotron design. In Sec. V we discuss our methodology and results obtained. Section VI summarizes this study.

II. FORMULATION OF THE PROBLEM

To illustrate the role of the two effects deteriorating the gyrotron efficiency which we discussed above, viz., the electron velocity spread and the spread in electron guiding center radii, let us assume that we can represent the gyrotron efficiency as

$$\eta = \eta_{ideal} f_1(\delta v_\perp) f_2(\Delta R_b). \tag{1}$$

In Eq. (1), η_{ideal} is the efficiency of an ideal gyrotron with both spreads being negligibly small and the functions in the right-hand side characterize the effect of the spread in electron orbital velocities $f_1(\delta v_\perp)$ and in guiding center radii $f_2(\Delta R_b)$. Next, we specify these functions.

First, let us carry out the qualitative treatment of the problem. Results of numerous studies of the dependence of the electron velocity spread on the cathode slant angle[10,13] allow us to approximate the dependence of the velocity spread on the angle θ between the emitter surface and the magnetic force line there by the parabola

$$\delta v_\perp = (\delta v_\perp)_{\min} + A\left(\theta - \theta^{opt}_{el-opt}\right)^2. \tag{2}$$

In Eq. (2), the minimum velocity spread $(\delta v_\perp)_{\min}$ corresponds to the optimal angle θ^{opt}_{el-opt}, which yields the minimal velocity spread found in the course of optimization of the gyrotron electron optics; this electron-optical optimal angle θ^{opt}_{el-opt} itself and the coefficient A are design specific. Below, we will estimate these values for a specific design.

Next, we characterize the dependence of the gyrotron efficiency on the electron velocity spread. For gyrotrons with a Gaussian axial structure of the resonator field, this dependence was calculated in Ref. 14 and reproduced in Ref. 11 by Fig. 4.7. These calculations, however, include one unrealistic assumption about the possibility to realize an arbitrarily high orbital-to-axial velocity ratio $\alpha = v_{\perp 0}/v_{z0}$ at small velocity spreads. If we take into account that due to various restrictions imposed by electron beam optics this ratio is typically limited by 1.5–2.0, the dependence calculated in Ref. 14 can be approximated by the linear function

$$f_1(\delta v_\perp) = 1 - 0.6\delta v_\perp. \tag{3}$$

Now we should characterize the dependence of the efficiency on the spread in the guiding center radii denoted by $f_2(\Delta R_b)$ in Eq. (1). As shown in Ref. 12, this function depends on gyrotron operating parameters, such as the external magnetic field, and, in principle, it can be approximated by the parabola

$$f_2 = 1 - B(\Delta R_b/\lambda)^2. \tag{4}$$

In Eq. (4), λ is the wavelength and the coefficient B is different for different operating regimes: for regimes with soft self-excitation it is smaller than for high-efficiency regimes with hard self-excitation. Typically, as follows from Ref. 12, when electrons are injected in an inner peak of the beam coupling to the cavity mode, the value of B varies from about 0.5 for regimes with soft self-excitation to about 1.1 for regimes with a high efficiency. Lastly, we should express the radial width of a beam in the interaction space via the width of the emitter d. In accordance with Busch's theorem and the adiabatic compression of the beam in the region between the gun and the resonator, this dependence can be given as

$$\Delta R_b = d\sin\theta/\sqrt{\alpha_B}. \tag{5}$$

In Eq. (5), $\alpha_B = B_{res}/B_{cath}$ is the magnetic compression ratio, i.e., the ratio of the magnetic field in the resonator to the magnetic field in the cathode region.

Substituting formulas (2)–(5) into (1) allows one to describe the effect of the angle θ on the gyrotron efficiency by the function

$$\Phi = \eta/\eta_{best} = \left[1 - 0.6\bar{A}(\theta - \theta^{opt}_{el-opt})^2\right] \times [1 - \bar{B}(\sin\theta)^2]. \tag{6}$$

In Eq. (6), $\eta_{best} = \eta_{ideal}[1 - 0.6(\delta v_\perp)_{\min}]$ is the best efficiency that can be achieved in a gyrotron with the minimum velocity spread and zero spread in electron guiding centers, and parameters $\bar{A}$ and $\bar{B}$ relate to previously introduced parameters A and B as $\bar{A} = A/[1 - 0.6(\delta v_\perp)_{\min}]$ and $\bar{B} = B[(d/\lambda)^2/\alpha_B]$. So, the optimal angle which yields the maximum efficiency with the account for the electron spread in guiding centers is the solution of the transcendental equation which follows from Eq. (6)

$$1.2\bar{A}(\theta_{opt} - \theta^{opt}_{el-opt})[1 - \bar{B}(\sin\theta_{opt})^2] + [1 - 0.6\bar{A}(\theta_{opt} - \theta^{opt}_{el-opt})^2]\bar{B}\sin 2\theta_{opt} = 0. \tag{7}$$

III. RESULTS

Results of simulations are shown in Fig. 1 as plots illustrating the dependence of the function Φ on the parameter $\bar{B}$ for several values of the parameter $\bar{A}$. These plots are shown for several values of the angle θ^{opt}_{el-opt}, which is the optimal angle providing the minimal velocity spread. Dashed lines indicate the functions $\Psi = \Phi(\theta_{opt}) - \Phi(\theta^{opt}_{el-opt})$, which characterize the benefits from optimization done with the account for the finite width of the emitter.

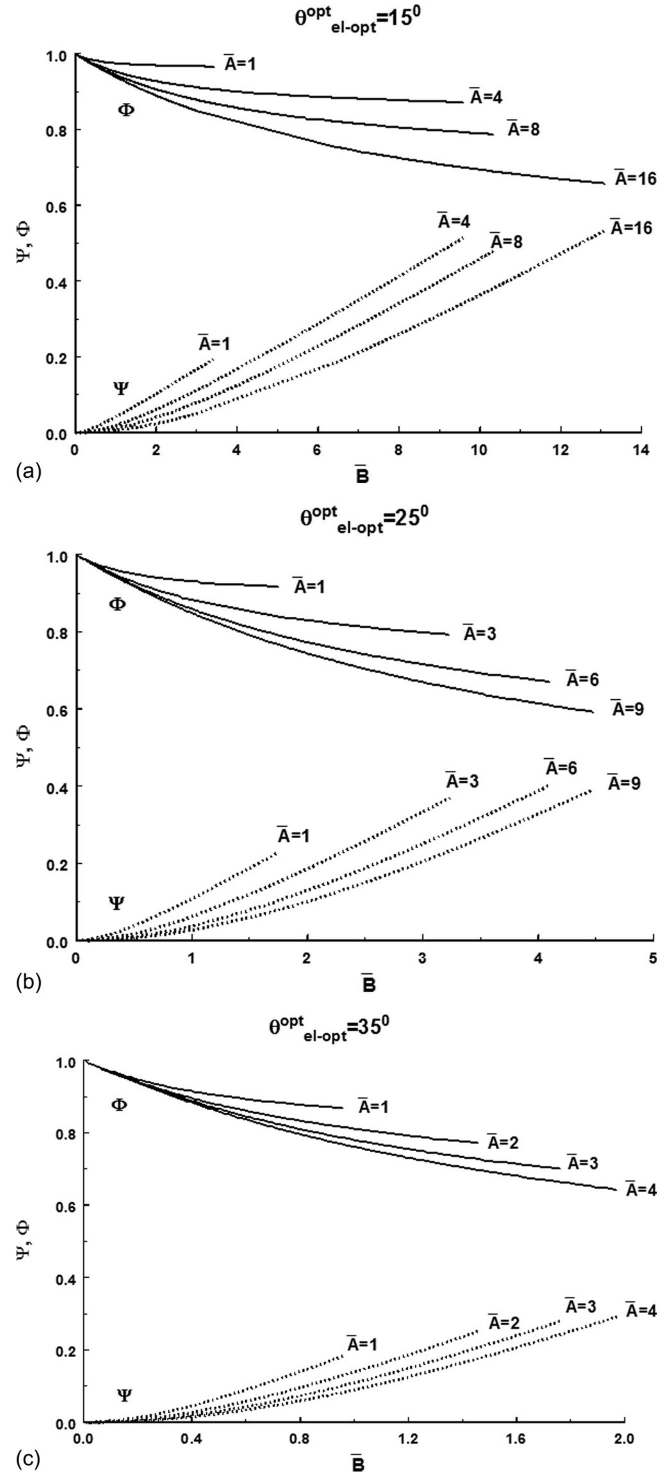

FIG. 1. Dependence of $\Phi = [1 - 0.6\bar{A}(\theta_{opt} - \theta^{opt}_{el-opt})^2] \times [1 - \bar{B}(\sin\theta_{opt})^2]$ (solid) and $\Psi = \Phi(\theta_{opt}) - \Phi(\theta^{opt}_{el-opt})$ (dashed) on $\bar{B}$ for several values of the parameter $\bar{A}$.

Corresponding dependences of optimal angles which yield the maximum efficiency in gyrotrons with finite width of emitters on the parameter $\bar{B}$ are shown in Fig. 2. As one can see, as the emitter width (parameter $\bar{B}$) increases, these

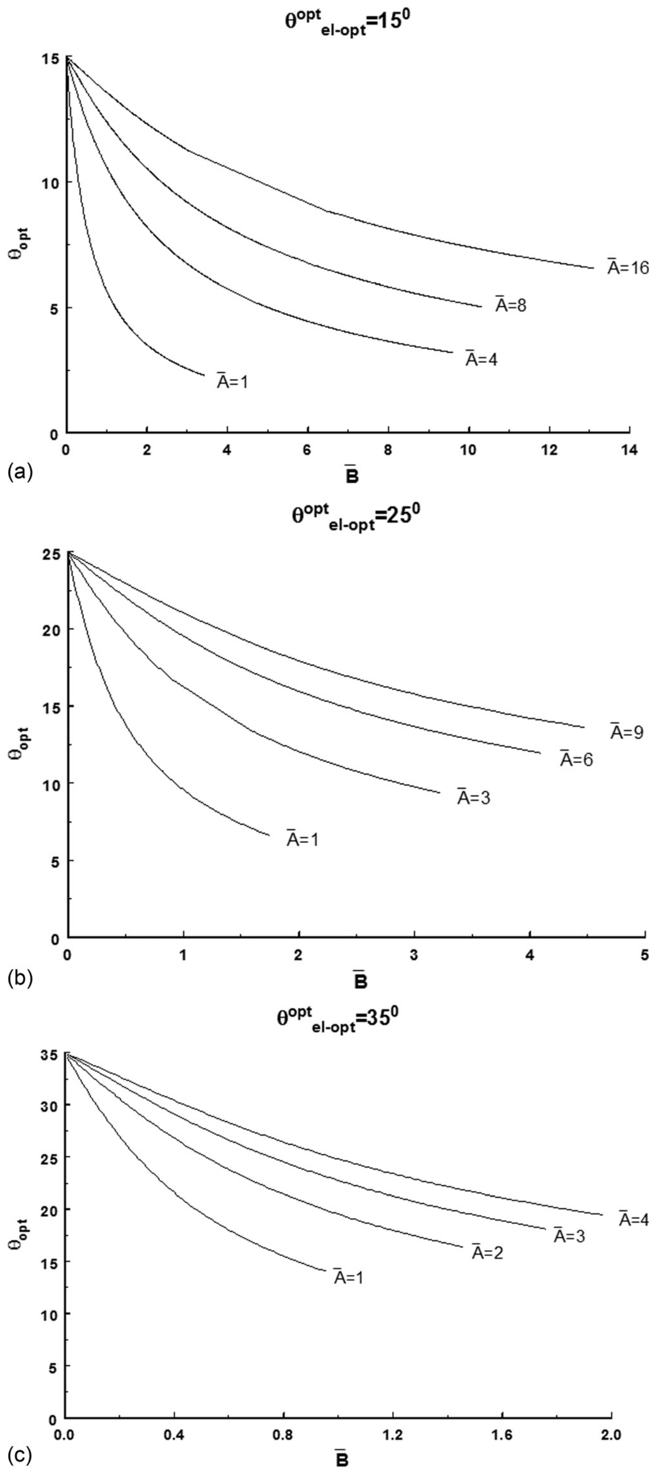

FIG. 2. θ_{opt} as a function of $\bar{B}$ for several values of $\bar{A}$ expressed in units of $(1/rad^2)$.

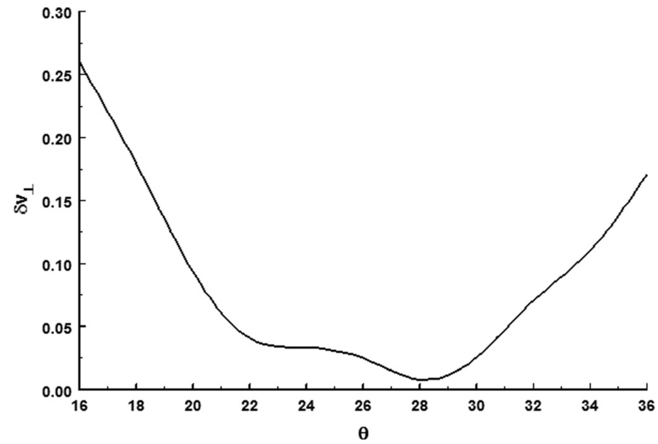

FIG. 3. Velocity spread as a function of θ.

angles θ_{opt} further depart from the optimal angles θ_{el-opt}^{opt}, which provide the minimal velocity spread.

IV. EXAMPLE

Consider as an example parameters of a 300 GHz gyrotron.[13] This gyrotron operates in the $TE_{14,2}$-mode. The width of the emitter is 3.2 mm and the magnetic compression factor $\alpha_B = B_{res}/B_{cath}$ is equal to 53.2. Thus, for the case of operation in the high-efficiency regime of hard self-excitation (where B, as mentioned above, is close to 1.1) we can estimate the parameter $\bar{B}$ for this gyrotron equal to 0.21.

Let us now evaluate the parameter $\bar{A}$. Results of calculations of the electron velocity spread as the function of the angle θ are shown in Fig. 3.

These data are based on calculations presented in Ref. 13 under the assumption that the beam orbital-to-axial velocity ratio is equal to 1.2. As follows from Fig. 3, the angle θ yielding the minimal velocity spread is close to $28^\circ = 0.49$ rad, and the corresponding minimal velocity spread is very small (about 0.007). Since we are interested in the dependence of the spread on the angle θ at $\theta < \theta_{el-opt}^{opt}$, in this range of angles the dependence shown in Fig. 3 can be approximated by a parabola (2) with $A \approx 3.475(1/\text{rad}^2)$. Correspondingly, our parameter $\bar{A}$ is close to $\bar{A} \approx 3.6(1/\text{rad}^2)$.

For these values of the parameters ($\bar{B} = 0.21$ and $\bar{A} \approx 3.6(1/\text{rad}^2)$) the functions Φ and Ψ shown in Fig. 1 are equal to 0.96 and 0.004, respectively, and the optimal angle θ_{opt} shown in Fig. 2 is equal to 25° instead of 28°. So, the deviation from optimal values found in the absence of this effect is relatively small. However, if we assume that for increasing the total beam current, the emitter width is doubled, this makes the value of the parameter $\bar{B}$ four times larger, and the effect becomes much more significant: in this case $\theta_{opt} = 21^\circ$.

It is interesting to compare these qualitative results with quantitative simulations. In general, the output efficiency relates to the interaction efficiency as

$$\eta_{out} = \frac{Q_{ohm}}{Q_{ohm} + Q_{dif}} \eta_{\text{int}}. \tag{8}$$

In Eq. (8), the relation between the ohmic and diffractive Q-factors defines the proportion between the power of outgoing radiation and the power of microwave losses in the circuit walls. The interaction efficiency in the case under study can be defined, in line with the definition given elsewhere,[13,14] as

$$\eta_{\text{int}} = \frac{1}{\Delta R_b} \int_{R_{b,\min}}^{R_{b,\max}} \left\{ \int_0^{v_0} v_{z0} \left(\frac{v_{\perp 0}}{v_0} \right)^2 f(R_b, v_{\perp 0}) \eta_{\perp}(R_b) dv_{\perp 0} \right\} dR_b. \tag{9}$$

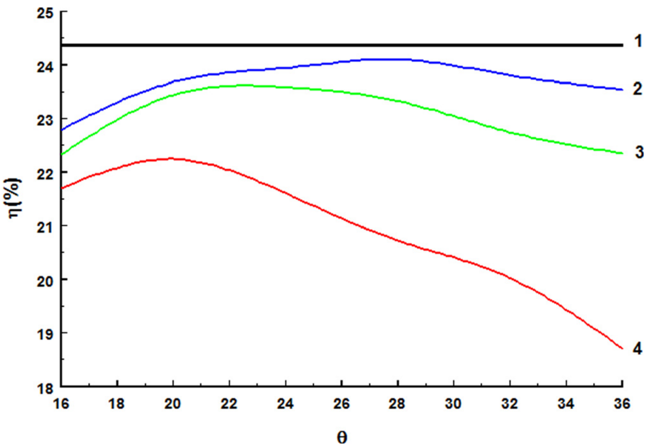

FIG. 4. Efficiency as a function of θ. Curve 1 (black line) shows the case of no spreads; curve 2 (blue line) shows the case of α spread for $d = 3.2$ mm; curve 3 (green line) shows the case of both spreads in R_b and α for $d = 3.2$ mm; and curve 4 (red line) shows the effect of both spreads for $d = 6.4$ mm.

In Eq. (9), $\eta_\perp$ is the perpendicular or orbital efficiency of a single beamlet with specified alpha-ratio and the guiding center radius. Equation (9) is written for a mono-energetic electron beam with spread in $\alpha = v_{\perp 0}/v_{z0}$ and in R_b. The function $f(R_b, v_{\perp 0})$ describes the electron distribution in the guiding center radii and the initial orbital velocities.

Simulations were done for a 300 GHz gyrotron mentioned above. This gyrotron is driven by a 65 kV, 10 A electron beam operating in the magnetic field of 11.4 T. The mean value of the guiding center radius is 2.42 mm, and the spread in the guiding center radii in the resonator is 0.40 mm. The ohmic and diffractive Q-factors are equal to 15 400 and 9100, respectively. Results of simulations performed for different angles are shown in Fig. 4.

As expected, in the absence of spread in beam radii (blue curve), the efficiency has its maximum at $\theta = 28°$, where α spread has a minimum. Inclusion of the spread in electron guiding centers (green and red curves) shifts the optimal angle θ_{opt} to the left which is consistent with the analytical results presented above.

V. DISCUSSION

In Sec. IV we illustrated the effects analyzed in Sec. III by using some available numerical data on a specific electron beam in a specific design. Strictly speaking, the analysis of the effect under study should be done as follows. It is necessary to start from considering several points along the emitter like those shown by Fig. 2.3 in Ref. 11. At each point we have several rays showing electrons emitted at slightly different angles that yields a spread in α for electrons gyrating about one magnetic force line. Let us emphasize that this spread depends on the axial coordinate of the emitting point. So, when we calculate the average efficiency of a gyrotron with the spread in α and in the guiding center radii, we should take this axial dependence into account and carry out optimization of gyrotron electron optics simultaneously with optimization of the circuit parameters for the most efficient beam-wave interaction.

Because of the absence of data on the velocity spread and α for each electron ray emitted from a specific point of the emitter, above we used the data for the spread in α assuming that this spread is the same for all emitting points.

It seems possible to formulate a simple "rule of thumb" which allows one to determine when the effect under study can be important. As follows from Ref. 12, when an electron beam is injected in the inner peak of the resonator field, the efficiency deterioration becomes significant when the beam thickness is equal or larger than one third of a wavelength

$$\Delta R_b = d \sin\theta / \sqrt{\alpha_B} \geq \lambda/3. \qquad (10)$$

The beam current emitted from an emitter of the width d is equal to $I_b = j_{cath} 2\pi R_e d$, where j_{cath} is the cathode loading and the emitter radius R_e relates to the radius of a beam injected in the caustic region of the operating mode in the resonator as $R_e = R_{caust}\sqrt{\alpha_B} \approx (m\lambda/2\pi)\sqrt{\alpha_B}$, where m is the azimuthal index of the mode. Substituting these simple formulas into Eq. (10) yields the following restriction on the choice of the angle allowing for neglecting the effect of the radial spread

$$\sin\theta_{opt} \leq \alpha_B m \frac{j_{cath}\lambda^2}{3I_b}. \qquad (11)$$

At present, there is a strong interest in developing 0.5 MW, 300 GHz gyrotrons for plasma experiments at the tokamak Ignitor.[15] Assuming that such a gyrotron operates in the $TE_{22,6}$-mode, the beam current equals 30 A, magnetic compression factor is 50, and the cathode loading is 3 A/cm^2, we derive from Eq. (11) that the effect of the radial spread becomes significant when the optimal angle of the cathode exceeds 22°.

VI. SUMMARY

In this study, the theory was developed which describes the optimization of gyrotron parameters when the tradeoff between α spread and R_b spread is taken into account. It is shown how this tradeoff influences the choice of the optimal value of the slant angle of the emitter. A simple "rule of thumb" is formulated which can be used to determine when this tradeoff should be taken into account.

ACKNOWLEDGMENTS

The work of G.S.N. was supported by the Office of Fusion Science of the US Department of Energy.

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