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Calculations of Starting Currents and Frequencies in Frequency-Tunable Gyrotrons

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Cold cavity and self-consistent formalisms for starting current and frequency calculations in frequency-tunable gyrotrons are summarized. Numerical solution schemes of the corresponding equations are discussed. A specific case is analyzed in detail.

1. Introduction

In recent years, much attention has been paid to the development of frequency-tunable low-power high-frequency gyrotrons.[1–6] Such gyrotrons can be used to study dynamic nuclear polarization, nuclear magnetic resonance, and the hyperfine split of positronium measurements. Here, the tuning scheme is based on the excitation of cavity modes that only differ by their number of axial variations on their field profile, which may lead to continuous tunability.

In this paper, we summarize cold cavity and self-consistent formalisms for calculating starting currents and frequencies. A comparison of the obtained results shows that they are very different and that for a reliable design the use of the self-consistent approach is mandatory.

2. Formalism

The gyrotron equations have been known for many years. Many researchers have made important contributions to deriving and refining these equations.[7–27) Nevertheless, we think that it is useful once again to present compact and concise formulations without dwelling on corresponding proofs. Most such formulations can be found in the monograph.[28]

2.1 Cold cavity formalism

In the cold cavity formalism, first the second-order differential equation for the rf field profile f has to be solved:

$$\frac{d^2 f}{d\zeta^2} + \gamma^2 f = 0, \tag{1}$$

where

$$\zeta = \frac{\beta_\perp^2 \omega_{\mathrm{cyc}}}{2\beta_\parallel c} \cdot z \tag{2}$$

is a dimensionless longitudinal coordinate and

$$\gamma = \frac{z}{\zeta}\sqrt{\frac{\omega^2}{c^2} - \frac{\nu^2}{R^2}} \tag{3}$$

is the generalized wave number. Here, $\beta_\perp = v_\perp/c$ and $\beta_\parallel = v_\parallel/c$ are the normalized perpendicular and longitudinal electron velocities, respectively, $\omega_{\mathrm{cyc}} = 2\pi 28 B/\gamma_{\mathrm{rel}}$ is the electron cyclotron frequency in GHz, B is the magnetic field in T, $\gamma_{\mathrm{rel}} = 1 + U/511$ is the relativistic factor, U is the acceleration voltage in kV, $\omega = \omega_{\mathrm{r}}(1 + i/Q_{\mathrm{D}})$ is the complex rf frequency, Q_{D} is the diffraction quality factor, ν is the mode eigenvalue, and R is the cavity radius. Equation (1) is supplemented by the following boundary conditions:

$$\frac{df}{d\zeta_{|\zeta=0}} = i\gamma f, \tag{4}$$

$$\frac{df}{d\zeta_{|\zeta=\zeta_{\mathrm{out}}}} = -i\gamma f. \tag{5}$$

Equation (1) is solved in such a way that the two unknown quantities, the real part ω_{r} of the rf frequency and the diffraction quality factor Q_{D}, are varied until the boundary condition eq. (5) is satisfied. Mathematically speaking, we face the problem of the minimization of a two-dimensional functional. This is a rather difficult problem because, in principle, there are many pairs of values of ω_{r} and Q_{D} for which the condition eq. (5) is "almost" satisfied. It is not easy to find the deepest minimum [the smallest discrepancy of eq. (5)]. Here, two approaches are possible. In the first approach, an initial estimation of the values of ω_{r} and Q_{D} is made for some minimization scheme, for example, the steepest descent gradient method. Of course, there is no guarantee that the "absolute minimum" of deviation from eq. (5) will be found at once. Sometimes many attempts are needed until a plausible absolute minimum is found. The second approach, used in the present study, is straightforward but very time-consuming. One solves eq. (1) on the $\omega_{\mathrm{r}} Q_{\mathrm{D}}$ grid and chooses that pair of ω_{r} and Q_{D} as a solution that leads to the smallest deviation from eq. (5). If the grid is fine enough, the second approach is more reliable.

Once eq. (1) is solved, we know the field profile f, the real part of the cavity resonance frequency ω_{r}, and Q_{D}.

In the second step of the cold cavity formalism one has to solve the electron motion equation:

$$\frac{dp}{d\zeta} + ip(\Delta + |p|^2 - 1) = if(p^*)^{n-1}G, \tag{6}$$

where p is the normalized electron orbital momentum, n is the harmonic number, and

$$\Delta = \frac{2}{\beta_\perp^2}\left(\frac{\omega_{\mathrm{r}} - n\omega_{\mathrm{cyc}}}{\omega_{\mathrm{r}}}\right) \tag{7}$$

is the frequency mismatch. The electron beam coupling to the rf field is given by the lengthy expression

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$$G = \left[0.47\times10^{-3}\cdot Q_D\cdot P_{out}\frac{J^2_{m\pm n}\left(\frac{\nu}{R_0}\cdot R_{el}\right)}{\gamma_{rel}U\eta_{el}\beta_\parallel\beta_\perp^{2(2-n)}(\nu^2-m^2)J_m^2(\nu)\int_0^{\zeta_{out}}|f(\zeta)|^2\,d\zeta}\right]^{1/2}\cdot\left(\frac{n^{n+1/2}}{2^n n!}\right), \quad (8)$$

where P_{out} is the output power,

$$\eta_{el} = \frac{\beta_\perp^2}{2(1-\gamma_{rel}^{-1})} \quad (9)$$

is the electron efficiency, m is the azimuthal wave number, R_0 is the cavity radius at the beginning of the straight section, and R_{el} is the electron beam radius. Equation (6) must be supplemented by the following initial condition at the entrance to the cavity:

$$p(0) = \exp(i\vartheta), \quad (10)$$

where ϑ is the electron phase ($0 \le \vartheta < 2\pi$). The orbital efficiency is given by

$$\eta_\perp = 1 - \frac{1}{2\pi}\int_0^{2\pi}|p(\zeta_{out})|^2\,d\vartheta. \quad (11)$$

The ohmic quality factor can be calculated as

$$Q_{ohm} = \frac{R_0}{\delta}\left(1-\frac{m^2}{\nu^2}\right), \quad (12)$$

where the skin depth δ is given by

$$\delta = \sqrt{\frac{\lambda}{\pi Z_0\sigma}}. \quad (13)$$

Here, λ is the wavelength in mm, $Z_0 = 377\,\Omega$ is the free-space impedance, and $\sigma = 57000\,(\Omega\cdot\text{mm})^{-1}$ is the wall conductivity for copper. It should be noted that at frequencies of approximately 200 GHz considered in what follows this conductivity is approximately 1.5 times lower owing to surface roughness. Finally, the output power can be calculated as

$$P_{out} = U\cdot I_A\cdot\eta_\perp\cdot\eta_{el}\cdot\frac{Q_{ohm}}{Q_{ohm}+Q_D}, \quad (14)$$

where I_A is current in amperes. The starting current is given by

$$I_{st} = \frac{\gamma_{rel}\beta_\parallel\beta_\perp^{2(2-n)}(\nu^2-m^2)J_m^2(\nu)\int_0^{\zeta_{out}}|f(\zeta)|^2\,d\zeta}{0.47\times10^{-3}Q_D J^2_{m\pm n}\left(\frac{\nu}{R_0}\cdot R_{el}\right)\Gamma}\times\left(\frac{2^n n!}{n^{n+1/2}}\right)^2, \quad (15)$$

where

$$\Gamma = -\left(n+\frac{\partial}{\partial\Delta}\right)\left|\int_0^{\zeta_{out}}f(\zeta)e^{in\Delta\zeta}\,d\zeta\right|^2. \quad (16)$$

2.2 Self-consistent formalism

In the self-consistent formalism, one has to simultaneously solve the following system of two differential equations:

$$\begin{cases}\dfrac{dp}{d\zeta} + ip(\Delta+|p|^2-1) = if(p^*)^{n-1}\\[2mm] \dfrac{d^2f}{d\zeta^2}+\gamma^2 f = I\cdot\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}p^n\,d\vartheta\end{cases}, \quad (17)$$

Table I. Frequencies and diffractive quality factors for the $TE_{0,3,q}$ mode with different axial indices q.

q	F (GHz)	Q_D
1	203.37	4467
2	203.93	1124
3	204.85	506

where the dimensionless current I is given by

$$I = 3.76\times10^{-3}\cdot I_A\cdot\beta_\parallel\beta_\perp^{2(n-4)}\cdot n^3 \times\left(\frac{n^n}{2^n n!}\right)^2\cdot\frac{J^2_{m\pm n}\left(\frac{\nu}{R_0}\cdot R_{el}\right)}{\gamma_{rel}\cdot J_m^2(\nu)\cdot(\nu^2-m^2)}. \quad (18)$$

In the self-consistent formalism, the rf frequency ω is a real quantity and one of the fitting parameters; the other fitting parameter is $\Re(f(0))$. These two parameters are varied as described above until the minimum deviation from the boundary condition eq. (5) is achieved. The diffraction quality factor Q_D is not a fitting parameter in the self-consistent formalism, but it can be evaluated at the end by

$$Q_D = \frac{8\beta_\parallel^2}{\beta_\perp^4}\cdot\frac{\int_0^{\zeta_{out}}|f|^2\,d\zeta}{I\cdot\eta_\perp}\cdot\left(\frac{\omega}{\omega_{cyc}}\right)^2. \quad (19)$$

3. Numerical Example

To illustrate the two formalism approaches, we have chosen the gyrotron FU CW V, which was developed for detection of the hyperfine transition of positronium.[5] This gyrotron operates in the $TE_{0,3}$ mode at the fundamental harmonic ($n = 1$).

The geometry of the cavity is as follows: angles $\theta_1 = 2.55°$, $\theta_2 = 0°$, and $\theta_3 = 6°$. The lengths of the sections are $L_1 = 4$ mm, $L_2 = 15$ mm, and $L_3 = 5$ mm. The radius of the cavity (at the straight section) is $R_0 = 2.389$ mm.

The frequencies and quality factors obtained in the cold cavity approximation are summarized in Table I. The field profile corresponding to the lowest axial index $q = 1$ is shown in Fig. 1.

This gyrotron is driven by an electron beam with a nominal current $I_{op} = 0.2$ A and an acceleration voltage $U_{opt} = 20$ kV, at which the pitch factor is $\alpha = 1.5$. The optimal electron beam radius providing the strongest coupling to the rf field is $R_{el} = 0.43$ mm. In Fig. 2, starting current is shown as a function of magnetic field.

In the cold cavity approximation, a correspondence between magnetic field and rf frequency can be found by means of a relatively complicated formalism that describes electron frequency pulling.[29] In the self-consistent formalism, rf frequency is one of the two fitting parameters and for the fixed magnetic field it is found "automatically". This is illustrated in Figs. 3 and 4. For the operating current $I_{op} = 0.2$ A, no oscillations were found beyond $B = 7.65$ T.

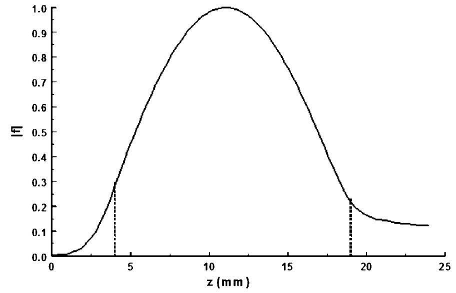

Fig. 1. Absolute value of the field profile of the $TE_{0,3,1}$ mode. The middle section of the cavity is between the two vertical dashed lines.

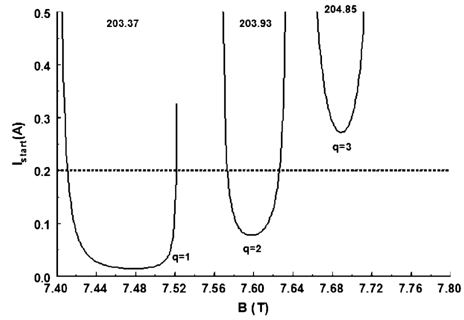

Fig. 2. Starting current as a function of magnetic field. The operating current $I_{\mathrm{op}} = 0.2\,\mathrm{A}$ is marked by the horizontal dashed line. The numbers inside the starting current curves correspond to the oscillation frequencies in GHz listed in Table I.

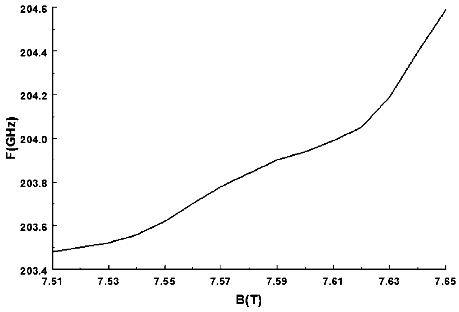

Fig. 3. Frequency as a function of magnetic field.

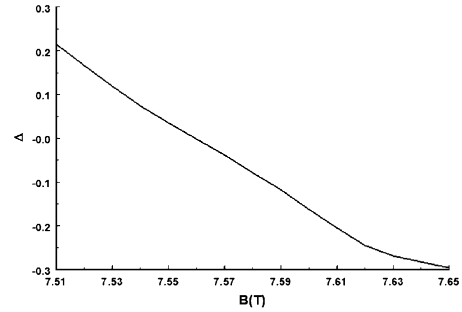

Fig. 4. Frequency mismatch as a function of magnetic field.

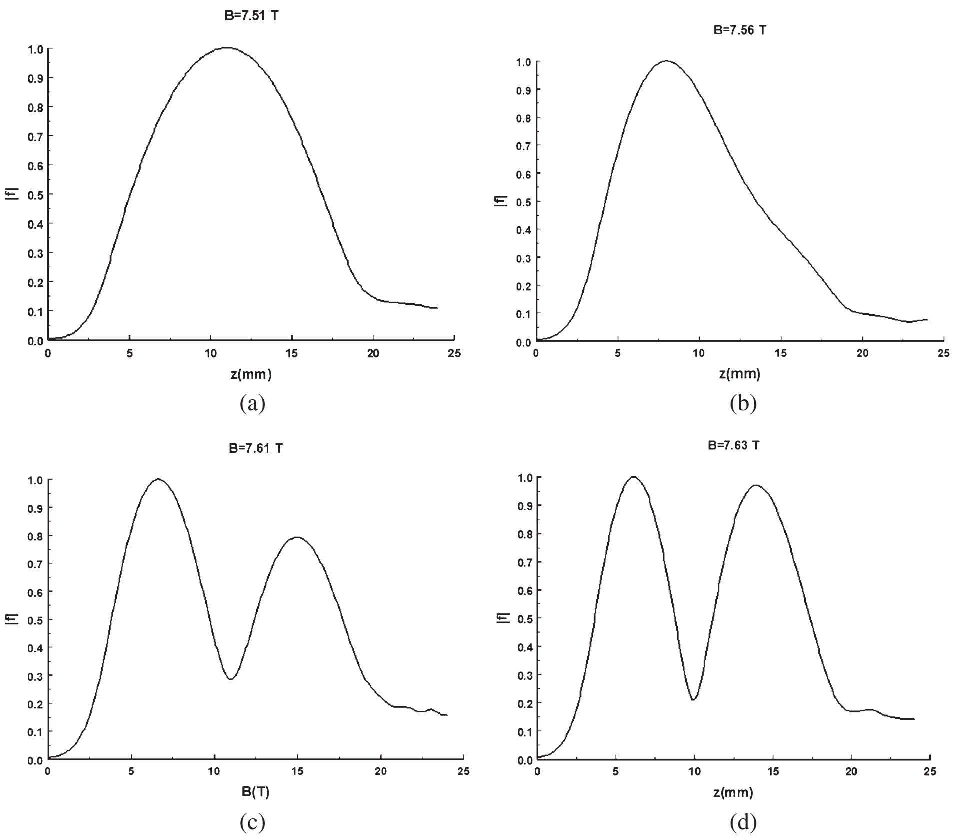

Fig. 5. Field profiles for selected magnetic fields. (a) $B = 7.51\,\mathrm{T}$, (b) $B = 7.56\,\mathrm{T}$, (c) $B = 7.61\,\mathrm{T}$, and (d) $B = 7.63\,\mathrm{T}$.

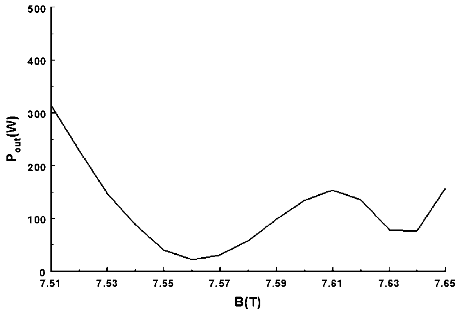

Fig. 6. Output power as a function of magnetic field. Here, $I_{op} = 0.2\,A$.

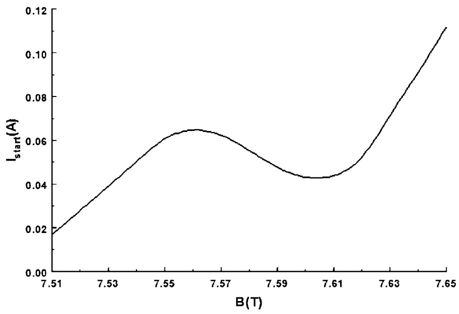

Fig. 7. Starting current as a function of magnetic field.

The corresponding field profiles for selected magnetic fields are shown in Figs. 5(a)–5(d) and the output power is shown in Fig. 6.

The self-consistent formalism also allows one to easily calculate starting current as a function of magnetic field. For a given magnetic field, one solves equations for different operating currents. The smallest current at which oscillations are excited can be called the starting current. The results of such calculations are shown in Fig. 7.

It is seen, that in contrast to the cold cavity case (Fig. 2), starting current is a smooth function of the field. This is the physical basis of continuous frequency tenability, which is related to the continuous deformation of the field profile.

4. Conclusions

By means of the self-consistent formalism, we have demonstrated that by changing magnetic field in the interval $7.51 \leq B \leq 7.65\,T$ it should be possible to continuously tune frequency in the interval $203.48 \leq F \leq 204.59\,GHz$. Here, output power also changes continuously, albeit not monotonically. Finally, it should be emphasized that by increasing magnetic field from 7.51 to 7.65 T, the dependences shown in Figs. 3 and 6 should be observed. However, by decreasing magnetic field from 7.65 to 7.51 T, different dependences are expected owing to hysteresis effects.[30] It would be interesting to study this experimentally.

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