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Field Formation in the Interaction Space of Gyrotrons

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Abstract For gyrotron applications in plasma installations, one of the most important factors is the gyrotron efficiency. To maximize the interaction efficiency, it is necessary not only to optimize such operating parameters as the magnetic field, beam voltage, and current but also the axial profile of the electromagnetic (EM) field in the interaction space. The present paper describes a study of the effect of the profile of an irregular waveguide serving as a resonator on the axial structure of the EM field. Specific attention is paid to the profile of the uptaper connecting the regular part of a resonator to the output waveguide. Conditions of applicability of the nonuniform string equation, which is widely used in gyrotron designs for finding the axial structure of the EM field, are discussed. Also discussed are the occurrence of reflections from a smooth uptaper and the analogy between the nonuniform string equation and the stationary Schrodinger equation.

Keywords Gyrotron · Open resonator · Potential well

1 Introduction

Gyrotrons are known as the most powerful sources of coherent millimeter- and submillimeter-wave radiation capable of continuous-wave operation [1–4]. For realizing the efficient operation at high-power levels it is desirable to optimize the length of the interaction space and the axial distribution of the electromagnetic (EM) field in it [5, 6]. For large-scale tokamaks and stellarators of next generations, the gyrotrons operating in the short-millimeter and sub-millimeter wavelength regions are required (see, e.g., [7, 8]). In such gyrotrons, for reducing the role of ohmic losses in efficiency degradation, it is necessary to use open resonators with minimum diffraction quality factors [5, 6]. In conventional gyrotrons, the resonators are formed by slightly irregular waveguides open in the axial direction toward collectors. The

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minimum diffractive quality factors can be realized in the case when such resonators are relatively short and do not have any cutoff narrowing at the output. The fundamentals of the theory of such resonators are described in [9].

In spite of many years of successful gyrotron development, it is still, however, not quite clear how one should determine the gyrotron interaction space and where the cyclotron resonant interaction between the waves propagating in a slightly irregular waveguide and the electrons gyrating in the external magnetic field decreasing after the solenoidal mid-plane terminates (we do not take here into account a separate issue of the aftercavity interaction addressed elsewhere [10–13].) As an example of the existing interest in this problem, let us mention a recent paper [14] devoted to definition of the effective cavity length. In recent years, the interest in this problem was also motivated by the development of gyrotrons for spectroscopic applications [15–17]. In such gyrotrons, the frequency can be continuously tunable by varying the external magnetic field leading to simultaneous variation of the axial profile of the cavity field. As a rule, increasing the external magnetic field causes transition from the mode with one axial variation to the mode with two variations and so on (see, e.g., [16]).

The purpose of the present study is to analyze the role which is played in formation of such modes by the profile of an irregular waveguide serving as a resonator. The paper is organized as follows. In Sect. 2, we discuss the nonuniform string equation and the conditions of its applicability. In Sect. 3, we analyze the diffractive quality factor Q_D in a relatively general way following the treatment done in [9], [18–20]. In this section, it is shown that the standard formula for the minimum diffractive Q widely used by many authors for evaluating the gyrotron operation gives a wrong scaling of the diffractive Q with the cavity length to the wavelength ratio: $Q_D \propto (L/\lambda)^2$. A more accurate is the scaling $Q_D \propto (L/\lambda)^3$ that follows from the formulas derived in [9] and [18]. It is also shown that the analytical formulas derived in [18] for the diffractive Q, which contain four parameters, can be reduced to the formulas containing one parameter only. In Sect. 4, we discuss the analogy between the nonuniform string equation and the Schrodinger equation and some specific features of solutions in the case of a resonator with an open end and minimum reflections. It is shown that in the absence of the cutoff neck in the output cross section of a resonator, the diffraction losses cannot be explained by the analogy with the tunneling effect present in the case of the potential well with a small finite barrier. In Sect. 5, we study the effect of profiles of the output and input tapers on the axial structure of the resonator field and the diffractive Q factor by considering as an example a resonator designed for an European 1 MW gyrotron [21] for the International Thermonuclear Reactor (ITER). In Sect. 6, we discuss the results obtained. Finally, in Sect. 7, we summarize the study.

2 Nonuniform String Equation

Let us consider an axially symmetric, slightly irregular waveguide whose radius R depends on the axial coordinate z. Assume that in a certain part of it, a $\mathrm{TE}_{m,p}$-wave can be excited near cutoff, i.e., in this part,

$$k_{s,\perp} \approx \omega / c \tag{1}$$

In Eq. (1), $k_{s,\perp}(z) = \nu_{m,p} / R(z)$ is the transverse wave number defined by the eigennumber $\nu_{m,p}$ of this mode and the variable radius $R(z)$ of the waveguide wall, the subindex "s" designates the sth mode, $\omega = 2\pi c/\lambda$ is the wave frequency, λ is the wavelength, and c is the speed of light.

Also, assume that the variation ΔR of the radius R at the length L is small

$$\Delta R/R << (\lambda/L)^2 \tag{2}$$

and the length L is sufficiently long

$$L^2 >> \lambda R \tag{3}$$

Under conditions shown in Eqs. (1)–(3), the axial distribution of the resonator field (whose time dependence is $e^{i\omega t}$) can be described by the function $f_s(z)$ which in the absence of an electron beam (a cold-cavity approximation) obeys the nonuniform string equation [22]

$$\frac{d^2 f_s}{dz^2} + k_{s,z}^2(\omega_s, z) f_s = 0 \tag{4}$$

In (4), $k_{s,z}(z) = \sqrt{(\omega/c)^2 - k_{s,\perp}^2(z)}$ is the axial wavenumber. The function $f_s(z)$ describes the axial profile of the magnetic field in the case of TE modes and that of the electric field in the case of TM modes. Note that Eq. (4) can also be used for describing the modes with a relatively large number of axial variations ($q>1$) as long as the length of each variation $L_q \approx L/q$ obeys the condition (3). When this condition is not valid, Eq. (4) should be replaced by a more general set of coupled wave equations describing the coupling between waves with different number of radial variations, which were formulated in [22–27].

Equation (4) should be supplemented by the boundary conditions. Typical gyrotron resonators are formed, as shown in Fig. 1, by a section of a straight waveguide of a constant radius connected with the down-tapered waveguide on the cathode side (on the left) and an uptapered waveguide on the collector side (on the right). At sufficiently large distances from the cylindrical section, where the axial wavenumber is large enough and the waveguide profile varies smoothly enough,

$$\left|\frac{dk_{s,z}}{dz}\right| << \left|k_{s,z}^2\right| \tag{5}$$

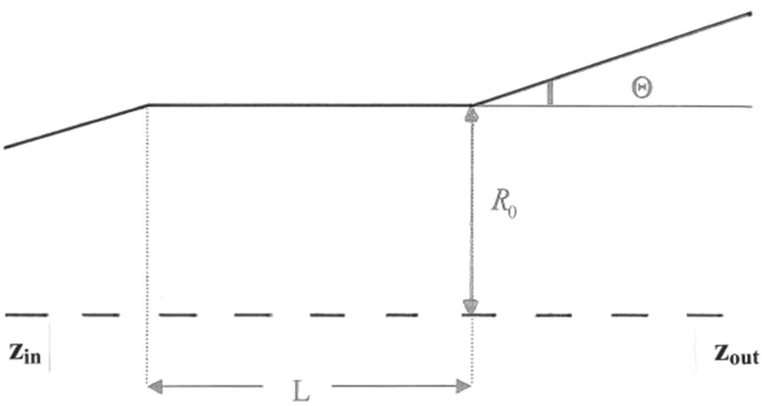

Fig. 1 Schematic of a straight waveguide connected with the linear uptaper

the solution of (4) can be written in the adiabatic WKB approximation as

$$f_s(z) = \frac{C_1}{\sqrt{k_{s,z}(z)}} e^{i\int k_{s,z}dz} + \frac{C_2}{\sqrt{k_{s,z}(z)}} e^{-i\int k_{s,z}dz} \tag{6}$$

Correspondingly, the boundary conditions (Sommerfeld conditions) in cross sections satisfying (5) can be given as the condition for the decaying field on the left

$$\left.\frac{df_s}{dz}\right|_{z_{in}} = \left|k_{s,z}\right|_{z_{in}} f_s(z_{in}) \tag{7}$$

and the condition for the outgoing radiation

$$\left.\frac{df_s}{dz}\right|_{z_{out}} = -ik_{s,z}(z_{out})f_s(z_{out}) \tag{8}$$

on the right.

Note that the condition (3) can be rewritten as the restriction on the Fresnel number introduced in [28, 29] for quasi-optical resonators as $N_F = a^2/\lambda d$ where a is the mirror radius and d is the distance between two mirrors. In gyrotron resonators operating in rotating $TE_{m,p}$ modes excited near cutoff, the role of the mirror radius is played by the effective resonator length and the distance between two successive reflections of the rays forming such modes from the walls is equal to $2R\sqrt{1-m^2/\nu_{m,p}^2}$. As explained in [6, 14], Eq. (3) can be rewritten as the condition $C_F >> 1$ for the equivalent Fresnel parameter

$$C_F = \pi\left(\frac{L_{eff}}{\lambda}\right)^2 \frac{1}{4\sqrt{\nu_{m,p}^2 - m^2}} \tag{9}$$

Note that in (9), the effective length L_{eff} discussed in [14] characterizes the profile of the resonator field and can be different from the length of the regular section L shown in Fig. 1, as will be discussed below.

3 Diffraction losses and quality factor

Let us start from deriving a simple formula describing the diffractive quality factor of a piece of a waveguide with open ends. By ignoring the wave reflections from open ends and denoting the waveguide length by L, we can start from a simple general formula for the diffractive Q factor $Q_D = \omega L/v_g$. Here, the group velocity can be represented as $v_g = c^2/v_{ph}$ and the phase velocity is $v_{ph} = \omega/k_z$ where for a standing wave with q axial variation the axial wavenumber is $k_z = q\pi/L$. Combining these simple formulas, one can easily get the expression for the minimum diffractive Q factor [6, 20, 30]:

$$Q_{D,\min} = \frac{4\pi}{q}\left(\frac{L}{\lambda}\right)^2 \tag{10}$$

In fact, however, the reflections from open ends are not equal to zero. The analysis of this issue important for open waveguides was carried out for potential wells in the quantum theory in [31] and then analyzed in details by many authors. These reflections can greatly increase the diffractive Q factor, and this is why we analyze this effect below.

The careful analysis of the diffractive Q factor carried out in [18] resulted in deriving an analytical formula for Q_D given by Eq. (42) there. That formula was derived for the case when a waveguide section of a constant radius, which serves as the main part of the resonator, is connected with the linear uptaper as shown in Fig. 1. The linearity of the uptaper allows one to carry the study analytically.

Let us use the parameter of the irregularity introduced in [9].

$$\xi = \nu_{m,p}^2 \left(L/D_0\right)^3 \tan\theta \tag{11}$$

In (11), $D_0=2R_0$ is the diameter of the central part of a waveguide and θ is the angle of tapering. Taking into account the condition (1), the parameter (11) can be rewritten as

$$\xi = \frac{\pi^3}{\nu_{m,p}} \left(\frac{L}{\lambda}\right)^3 \tan\theta \tag{12}$$

In these notations, one can rewrite Eq. (42) of [18] as the ratio of the diffractive Q defined in the presence of reflections to the minimum diffractive Q given above by (10):

$$\frac{Q_D}{Q_{D,\min}} = \frac{\sqrt{\exp\left(-1.886\frac{q\pi}{2\xi^{1/3}}\right)}}{1-\exp\left(-1.886\frac{q\pi}{2\xi^{1/3}}\right)} \tag{13}$$

In the case of very large values of ξ, Eq. (13) reduces to a much simpler formula

$$\frac{Q_D}{Q_{D,\min}} = 0.33757\frac{1}{q}\xi^{1/3} \tag{14}$$

Note that Eq. (14) corresponds to Eq. (43) in [18]. Equation (14) is essentially the same as the formula given in [9]. As follows from (14), the diffractive Q (Q_D) of such a resonator with an uptaper scales with the ratio L/λ as $Q_D \propto (L/\lambda)^3$, while the minimum Q_D given by (10) scales as $Q_{D,\min} \propto (L/\lambda)^2$. Note that we cannot use Eqs. (13) and (14) for considering a limiting case of a vanishingly small value of the parameter ξ because when the angle of uptapering θ in (12) goes to zero, we get a semi-infinite waveguide in which the length of the straight section is infinitely long.

In [18], the dependence of the diffractive Q on the axial index q was shown for some specific cavities and specific modes. In fact, however, Eqs. (13) and (14) allow one to carry out a general analysis of the dependence of the diffractive Q or the ratio $Q_D/Q_{D,\min}$ on the parameter of the irregularity ξ. These dependencies are shown for the first three axial indices (q=1, 2, 3) in Fig. 2; solid lines correspond to the general formula (13); dashed lines correspond to the asymptotic case of large values of ξ given by (14). The circles and squares show the Q ratios for the modes with one and two axial variations, respectively, for specific examples discussed in Sect. 5. As one can see, results of calculations yield the Q ratios much higher than the analytical prediction. In turn, the analytical predictions are rather close and the asymptotic formula (14) predicts a little higher value of Q than the general dependence given by (13). A possible reason for this discrepancy is discussed in Sects. 6 and 7.

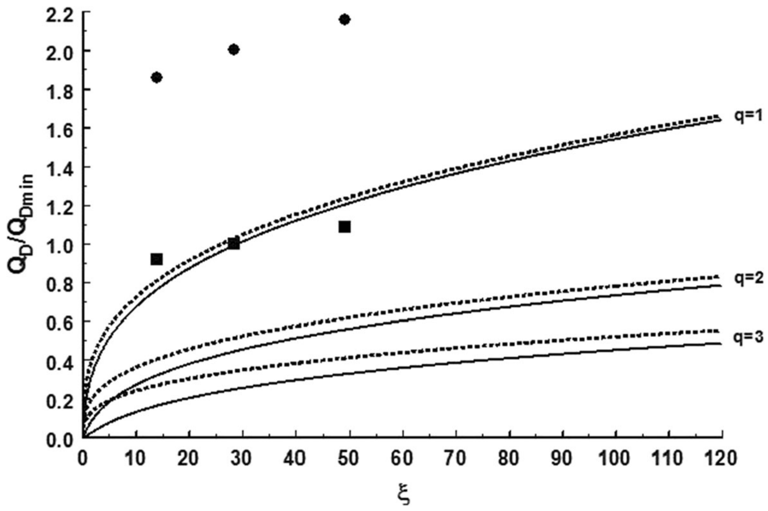

Fig. 2 Dependencies of the ratio $Q_D/Q_{D,\min}$ of the first three axial modes on the parameter of irregularity ξ defined by a general formula (13) (*solid lines*) and by an asymptotic formula (14) (*dashed lines*). *Circles and squares* show results of simulations described in Sect. 5 for $q=1$ and $q=2$, respectively

4 Parallel Between the Nonuniform String and Stationary Schrodinger Equations

As noted in [9], Eq. (4) is identical in form with the stationary Schrodinger equation (see, e.g., [31]) for a particle in a potential well with the profile $U(z)=k_{\perp}^{2}(z)$ corresponding to the profile of transverse wavenumbers in an irregular waveguide. In [9], the parallel between these two equations was discussed for the case when a resonator has a small local neck at the open collector side; that neck provides some reflections of the wave, but at the same time makes the wave tunneling through such a local barrier possible (as known, this tunneling is similar to the quantum tunneling causing the electron field emission process analyzed by Fowler and Nordheim [32] and many others). The height and the length of this barrier define the diffractive losses of the wave and, hence, its diffractive quality factor.

Less intuitive is consideration of a waveguide where such a neck is absent as the one shown in Fig. 1. A similar waveguide is used as an open resonator in the KIT design of a 1 MW, 170 GHz gyrotron for ITER which will be discussed in the next section. The profiles of this resonator (the original one together with its modifications considered in Sect. 5) is shown in Fig. 3.

The potential "well" $U(z)=\kappa^{2}(z)=\nu_{m,p}^{2}/R^{2}(z)$ corresponding to the original profile of this resonator is shown in Fig. 4. Here localization of the field inside a straight section of a waveguide (a flat portion of the well shown in Fig. 4) can be caused by some reflections from the cross section where an uptaper is connected to the straight section. These reflections cannot be explained by using the ray representation of the waves propagating in slightly irregular waveguides, but can be explained by considering the waves which experience a certain diffusion resulting in the reflection of some wave power from any irregularity of the waveguide surface [28]. The closer is the wave propagation to the cutoff, the stronger is the effect of irregularities on the wave reflection. Note that practically the same problem occurs in the case of neutrons propagating in the potential well formed by the Earth gravitational field and an additional horizontal partially reflecting mirror [33, 34]. The wave functions of the quantum states of neutrons in such wells are also described by the Schrodinger

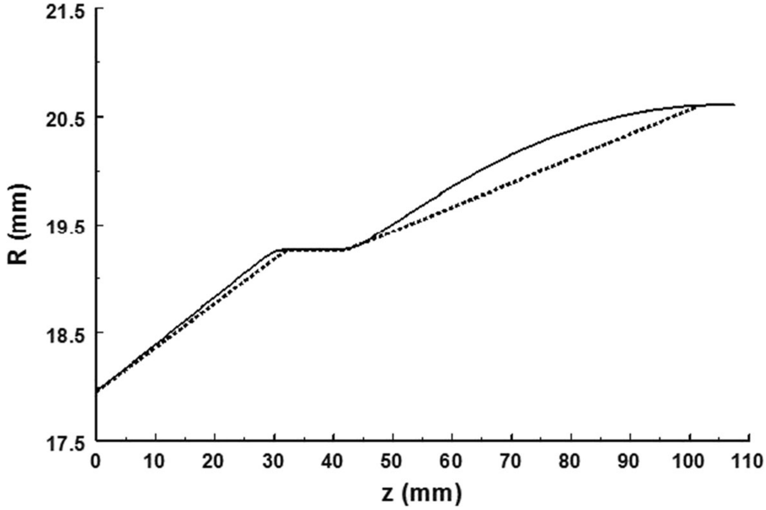

Fig. 3 Resonator profile of the 1 MW, European gyrotron for ITER with different uptapers. The original profile of the KIT resonator is shown by a *solid line*; the linear profile discussed below is shown by the *dotted line*

equation with the boundary conditions similar to (7) and (8). Note that a set of functions describing solutions of this problem is incomplete and their orthogonality is not proven [28].

5 Cold-Cavity Analysis of the Effect of Uptapering on the Resonator of a European 1 MW Gyrotron for ITER

Below, we consider as a base model the resonator whose straight section and cutoff narrowing on the cathode side are the same as in the European 1 MW gyrotron for ITER designed for

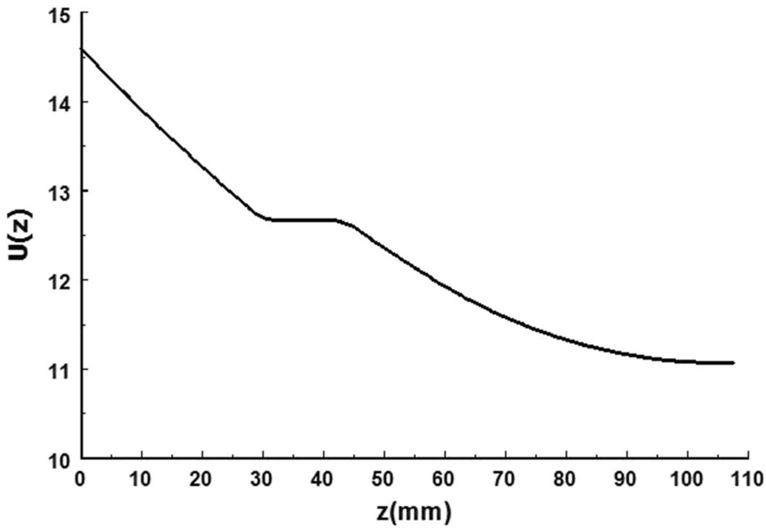

Fig. 4 Potential well corresponding to a resonator with the original uptaper shown in Fig. 3

operating in the $TE_{32,9}$ mode at the 170-GHz frequency [21]. The original profile of this resonator was shown in many papers (see, e.g., [14, 21]). In our analysis below, its uptaper is replaced by the linear uptaper because the purpose of this study is to compare results of calculations with predictions of the analytical theory developed for open resonators with linear uptapers. This profile is shown in Fig. 3 where the original profile is shown by the solid line, while the linear profile is shown by the dashed line. Axial distributions of the mode with one axial variation (q=1) in these resonators are shown in Fig. 5.

Corresponding values of quality factors are equal to $Q_{D,orig}$=1026, $Q_{D,lin}$=811, and $Q_{D,\min}$=403.5. It is interesting to note that in the original resonator, it was impossible to find any solution with more than one axial variation that demonstrates the efficient axial mode selection in it.

In order to compare some numerical results with described above analytical predictions, we also considered three cavities with the same linear uptaper, but longer straight sections. The quality factors of the modes with one and two axial variations in these cavities are given in Table 1 where also the corresponding values of the parameter ξ are shown. The values of this parameter ξ in Table 1 were calculated for the 170-GHz frequency, the operating mode $TE_{32,9}$ (the eigennumber $\nu_{32,9}$=68.56) and a given angle of uptapering 1.29°.

The ratio of these Q factors is shown in Fig. 2 that reveals the discrepancy between the numerical results and analytical predictions which is discussed below. Note that the changes in Q factors which can be tracked with the data in Table 1 correspond to the power dependence of diffraction quality factors intermediate between dependencies given by Eq. (10) [$Q_{D,\min}\propto(L/\lambda)^2$] and Eq. (14) [$Q_D\propto(L/\lambda)^3$]: as follows from Table 1, $Q_D\propto(L/\lambda)^\beta$ with $2.4<\beta<2.5$.

Axial structures of modes with one and two axial variations for some of the cases presented in Table 1 are shown in Fig. 6. Here, the first two figures illustrate the role of the uptapering angle on the axial structure of these two modes. The last figure (Fig. 6c) illustrates the effect of the length of the straight section.

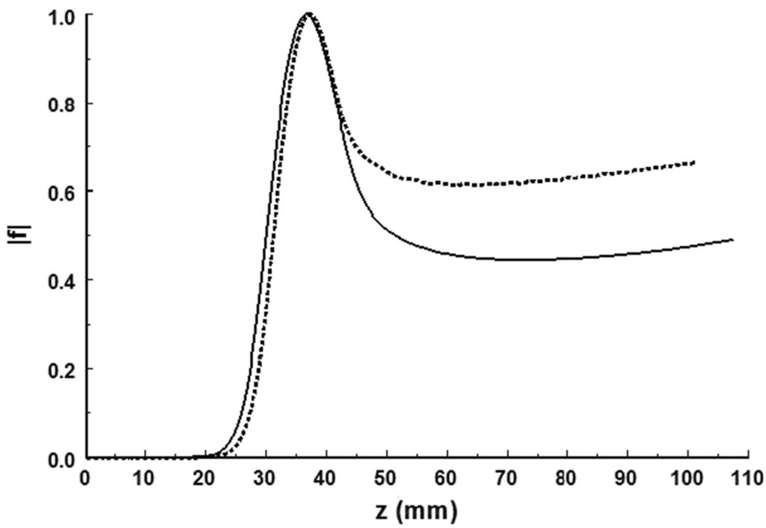

Fig. 5 Axial profiles of the q=1 mode in the original version (*solid*) of the KIT resonator and the one with the linear uptaper (*dashed*)

Table 1 Quality factors of modes with q=1 and q=2 in resonator with linear uptaper

$L(mm)$	ξ	$Q_{D,\min}(q=1)$	$Q_D(q=1)$	$Q_{D,\min}(q=2)$	$Q_D(q=2)$
20	14.06	1614	3004	807	746
25	28.42	2521	5044	1261	1265
30	49.11	3630	7856	1815	1972

For analyzing the role of the input taper, we replaced a smooth input taper with the 2.34° angle by a step-profile section (the radius of the input section 19.0 mm). The length of the straight section in both cases was the same (25 mm), and the angle of the linear uptapering in both cases was the same as well (1.29°). While in the case of a smooth input taper the diffractive Q was equal to 5044, in the case of a step-profile input taper, the value of this Q is equal to 4113, i.e., it dropped by about 20 %.

6 Discussion

Results presented above reveal the importance of waveguide taperings on the formation of axial structure of the resonator field in gyrotrons. In our study, we focused on the effect of the waveguide uptaper on the collector side. The down-taper on the cathode side not only affects the diffractive Q but is also important for the interaction efficiency because this down-taper is

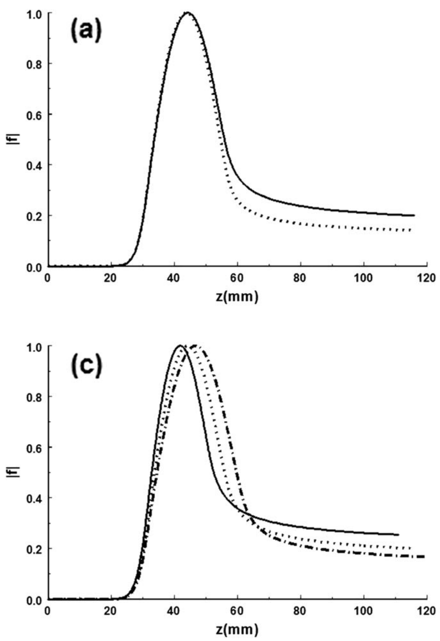

Fig. 6 Axial structures of the modes with one and two axial variations: **a** the mode with q=1, L=25 mm, a linear up taper with two angles of tapering 3° (*solid line*) and 1.29° (*dotted line*); **b** the mode with q=2 in the same resonator; **c** the mode with q=1 in resonators of a different length L=20 mm (*solid line*), L=25 mm (*dotted line*), L=30 mm (*dash-dotted line*) in the case of a linear uptaper with a 1.29° angle

important for formation of a "cathode tail" of the function $f_s(z)$. That tail is responsible for initial modulation of electron energies which causes subsequent formation of a compact electron bunch from electrons uniformly distributed in gyrophases at the entrance. The quality of this bunch defines the gyrotron interaction efficiency (see, e.g., [5, 6]).

Results presented in Fig. 2 showed a significant discrepancy between analytical predictions and numerical simulations, while in the previous study [18], there was an excellent agreement between the two. One possible explanation for this discrepancy can be based on the fact that the Fresnel parameter for a 170 GHz, 1 MW gyrotron for ITER is rather small, while for a resonator studied in [18], it was rather large. For our resonator with a 10-mm-long straight section that operates at the 170-GHz frequency in the $TE_{32,9}$ mode ($\nu_{32,9}$=68.56), as follows from Eq. (9), C_F=0.414, whereas the applicability of our theory is, strictly speaking, limited by the condition C_F>>1. It also deserves some attention the fact that the data presented in Table 1 yield the power dependence of the diffractive Q on the cavity length to the wavelength ratio intermediate between the known dependences of the minimum diffractive Q [Eq. (10)] and asymptotic analytical formula (14): $Q_D \propto (L/\lambda)^3$.

Much more accurate description of the processes in high-power gyrotrons can be achieved by using the self-consistent theory which takes into account the effect of an electron beam on the distribution of the resonator field. The stationary and nonstationary self-consistent theories of this sort were first formulated in [30, 35], respectively, and then developed further by many authors. Results of corresponding studies confirm that the axial structure of a resonator field in gyrotrons with low diffractive Q factors (close to $Q_{D,\min}$) strongly depend on gyrotron operating parameters, in particular, on the value of the external magnetic field [30]. Results of calculations carried out by using the self-consistent, nonstationary code MAGY [27] confirmed [36] that in the nonstationary regimes, the axial structure can strongly vary with time even when, for a certain time interval (within 100 ns), all gyrotron parameters are kept constant. Note that a typical time of variations in the field axial structure (about 5–10 ns) is much greater than the electron transit time through the resonator (about 0.1 ns). Therefore, the assumption about the smallness of the electron transit time in comparison with the cavity fill time used in MAGY as well as in many other codes is valid in our case and, correspondingly, there is no reason to take into account field modifications during the electron transit time, which were considered in [37–40].

7 Summary

It is shown that the diffractive Q (Q_D) of open cavities used in gyrotrons cannot be correctly evaluated by using a simple formula for the diffractive Q of a waveguide section open at both ends, as it was done by many authors previously. Instead, one can use either the analytical asymptotic formula derived in [9, 18] or a more accurate formula also derived in [18]. Our numerical results reveal, however, that for MW class gyrotrons all these formulas have a limited applicability. Nevertheless, the formulas can be used for evaluation of optimal parameters of gyrotron designs. In this regard, it should be noted that these formulas describe the dependence of diffractive losses not only on the cavity length to the wavelength ratio but also on the angle of tapering that yields more flexibility in the gyrotron design in comparison with the case of using an overly simplified formula for $Q_{D,\min}$. As shown in [18], in long resonators with a relatively large Fresnel parameter, there was a good agreement between analytical and numerical results. However, in the case of MW class gyrotrons with cavities having a relatively small Fresnel parameter, there is a significant discrepancy between analytical and numerical results.

Another issue deserving some attention is the analysis of reflections in an open cavity with a very smooth uptapering. Clearly, the ray representation of the outgoing radiation does not allow one to describe these reflections, so a more accurate analysis taking into account the diffusion of waves propagating near cutoff should be performed. In general, the present paper demonstrates that even in such a well studied area of the gyrotron theory as a cold-cavity theory of open resonators there are some issues which deserve further consideration.

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