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Start-up scenario of a high-power pulsed gyrotron for 300 GHz band collective Thomson scattering diagnostics in the large helical device

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We present results of theoretical study of mode competition during the voltage rise of a 300-kW, 300-GHz gyrotron operating in the TE22,2,1 mode. Simulations tracking eight competing modes show that, with a proper choice of the magnetic field, stable excitation of the operating mode can be realized, despite the presence of parasitic modes in the resonator spectrum. A finite voltage rise time, 1 kV/4 ns referred to as the slow voltage rise case, is taken into account to simulate realistically the experimental condition. Simulation results with the finite voltage rise time are in good agreement with the experimental test, in which the gyrotron demonstrated reliable operation at power levels up to 300 kW. Moreover, interesting phenomena are observed. Along with voltage rise, the oscillation manner changes from backward wave oscillation to gyrotron oscillation. In the range of the magnetic field lower than the magnetic field strength at which the TE22,2 mode attains to the maximum power, mode competition with the TE21,2 mode takes place although many other competing modes exist in between the two modes. In addition to the slow voltage rise case, the fast voltage rise case, 10 kV/4 ns, and the instant voltage rise case are considered. For these cases, simulations also predict stable oscillation of the TE22,2 mode with the same power level with the slow voltage rise case. This indicates that stable oscillations of the TE22,2 mode can be obtained in a wide range of the voltage rise time. © 2016 AIP Publishing LLC [http://dx.doi.org/10.1063/1.4941703]

I. INTRODUCTION

The gyrotron is a source capable of producing high power levels at millimeter–wave frequencies for many applications, including collective Thomson scattering (CTS) diagnostics in magnetic fusion devices. A 300 GHz band high power gyrotron is under development for application to the power source of the CTS diagnostics on the large helical device.1–3 This gyrotron is operated in a pulse mode and the start-up phase of each pulse passes a time-dependent variation of operating conditions before it reaches its steady-state operating point. It is necessary to excite the operating mode before parasitic modes and avoid the situation of mode cooperation when the presence of the desired operating mode decreases the starting current of a parasitic mode. The TE22,2 mode was chosen because of isolation from neighboring modes along with the design thought about the heat load of the cavity. In the design process of the 300 GHz gyrotron, instant rise of the cathode voltage was assumed for simplicity and 300 kW-level single mode oscillation of the TE22,2 mode was predicted. However, since, in the real experiment, the voltage rises with a finite time. Thus, it is necessary to carry out simulations taking into account the finite rise time. This paper is written with an attempt to understand the influence of the speed of voltage rise on mode competition.

The start-up scenario depends on the rise time of the operating voltage. Theoretical studies of start-up scenarios have been performed in numerous papers, for example, Refs. 4–8. The self-consistently calculated axial field profile was taken into account in Ref. 6. The voltage rise time is rather long for high power gyrotrons used in fusion experiments, and a sequence of mode transition is often predicted. However, when the voltage rise time is short enough, the mode competition shows a different picture. In Ref. 7, a rather fast voltage rise time case was examined for a coaxial 2 MW, 170 GHz ITER gyrotron with a simulation code. The simulation calculation has shown that the shortening of the voltage rise time to about 1 µs can eliminate excitation of parasitic modes during the start-up phase even for a gyrotron with a diode-type electron gun. One microsecond is much longer than the cavity decay time of a few ns typical to high power gyrotrons with frequencies around 100 GHz and even more than ten times as long as the grow time to MW level from a noise power. However, realization of a voltage rise time of 1 µs as an effectively fast voltage rise time is still difficult for power supplies used for real ITER level gyrotrons.

The present paper provides a new case study corresponding to the experimental condition. A semiconductor fast switching device is used for the power supply of the 300 GHz gyrotron for CTS diagnostics. The present simulation calculations stand for the real voltage rise time. The operating voltage rises with a rather short time, 1 kV/4 ns. In addition, mode competition is calculated for various magnetic fields. In particular, the time evolution of the TE22,2 mode is examined in detail around the low field side of the oscillation region of the TE22,2 mode where the highest power of the TE22,2 mode is expected. Experimentally, stable single mode oscillation at nearly 300 kW has been confirmed for the above rise time.9 The present study provides the foundation of the experiment.

Other aspect of the present study is the interaction between whispering gallery (WG) modes TE\textsubscript{m,n} such that
Strong interaction between WG modes usually occurs for $\Delta m = \pm 1$. While, in Ref. 4, start-up scenario was considered with a finite voltage rise time as well as instant rise, a fixed axial field profile was assumed. Simulations in the present paper self-consistently calculate the time evolutions of the axial field profile. The TE$_{22,2}$ mode possibly competes with the TE$_{21,2}$ mode in the region of the magnetic field where its highest power is attained. Mode hopping between the two modes is observed. Other possible competing modes between the TE$_{22,2}$ mode and the TE$_{21,2}$ mode are also included in simulation calculations and effects of these modes are examined. The simulation calculations show that stable high power oscillation of the TE$_{22,2}$ mode is obtained without sophisticated start-up

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FIG. 1. Absolute values of mode amplitudes at the cavity exit as a function of time and voltage for different magnetic fields.
II. GYROTRON CAVITY GEOMETRY, OSCILLATING MODES AND OPERATING PARAMETERS

The cavity profile has been chosen as follows. The angles are: $\theta_1 = 1.78^\circ$, $\theta_2 = 0^\circ$, and $\theta_3 = 3^\circ$. The lengths of the sections are: $L_1 = 5 \text{ mm}$, $L_2 = 8 \text{ mm}$, and $L_3 = 5 \text{ mm}$. The radius of the cavity (at the straight section) is $R_0 = 4.711 \text{ mm}$. In Table I, the cold frequencies and diffraction quality factors are shown for the operating mode $TE_{22,2}$ and its potential competitors. The $TE_{22,2}$ mode was chosen based on the same design concept of the prototype gyrotron and considering the heat load on the cavity surface. The heat load density is less than $2 \text{ kW/cm}^2$ for $300 \text{ kW}$ with a duty ratio up to 10%. This duty ratio is sufficient as the power rise case also gives a basis of a future gyrotron to generate a very short pulse train for new applications such as nonlinear THz spectroscopy.

The paper is organized as follows. Section II describes the gyrotron cavity geometry, oscillating modes, and operating parameters. Section III contains the results of mode competition calculations in the case of a slow voltage rise. The results obtained with the fast voltage rise and the instant voltage rise are presented in Section IV. Then, Section V presents the conclusions from the study.

In this gyrotron, a triode gun is used with the following parameters: the magnetic compression $b = 46.17$, the distance between the cathode and the anode $d = 7 \text{ mm}$, the cathode angle $\theta_c = 33.5^\circ$, and the cathode radius $R_c = 24.884 \text{ mm}$. This radius stands for the center of the emission belt. Although the electron gun has been carefully designed with the E-gun code to form a laminar electron beam, the velocity pitch factor is evaluated with an analytic expression in the simulation calculations. The dimensionless perpendicular electron velocity is given by the standard expression

$$\beta_p = \frac{1}{\gamma c} b^{3/2} \frac{(\cos \theta_c)^2}{\beta_1^2 / B \ln \left[ 1 + (d \cos \theta_c) / R_c \right]} U_{mod}.$$  

Here $\gamma = 1 + U_{cat} / 511$, $B$ is the magnetic field in the cavity, and $U_{mod}$ is the modulation voltage. In this particular gun, the modulation voltage is fixed at a ratio to the cathode voltage $U_{mod} = 0.464 \cdot U_{cat}$ by using a resistive divider. This ratio is constant throughout the pulse width. This situation is similar to that in a diode-type electron gun as long as the voltage rise during each pulse width is considered [see Fig. 5.6 in Ref. 13]. As shown later, this simple situation is enough for the start-up of the $TE_{22,2}$ mode to the maximum power. It is rather advantageous for the operation of a diagnostic-use gyrotron. The purpose of the triode gun is not to realize a sophisticated anode voltage evolution scenario but to optimize gun operation in a wide range of parameters such as the magnetic field, the beam voltage, and current and even for different modes with different frequencies. The parallel dimensionless electron velocity is $\beta_{||} = \sqrt{1 - 1 / \gamma^2} - \beta_p^2$ and the pitch factor is $\alpha = \beta_{||} / \beta_p$.

The operating current $I = 15 \text{ A}$ and the steady state operating voltage $U_{cat} = 65 \text{ kV}$. The frequency mismatch is defined as usual: $\Delta = \frac{2}{\beta_p} (\omega - \omega_{yc}) / \omega_{yc}$, where $\omega_{yc} = 2\pi \cdot 28B / \gamma$. Another important operating parameter is the voltage rise time. In this gyrotron, the power supply is rather fast near $0.25 \text{ kV/ns}$. Nevertheless, we call it slow in comparison with the future power supply near $2.5 \text{ kV/ns}$.

The cavity decay time $\tau_d = Q / \omega$ is about 1 ns for $Q = 2000$ and $\omega = 2\pi \cdot 300 \text{ GHz}$. The electron transit time through the cavity $\tau_p = L_d / c \beta_{||}$ is shorter than 0.1 ns for $\beta_{||} \sim 0.3$ corresponding to the present operation condition. Therefore, the condition $\tau_p \ll \tau_d$ safely holds. The normalized cavity length $\mu$ is about 11 and the normalized current $I_0$ is about 0.05 for the standard definition of $\mu$ and $I_0$. Although the rather fast voltage rise 0.25 kV/ns corresponds to the real experiment as mentioned already, 100 ns is necessary from 40 kV to 65 kV. The time evolutions of each mode
are calculated during this time period, which is much longer than $\tau_d$. This point is later discussed.

III. MODE COMPETITION CALCULATIONS IN THE CASE OF SLOW VOLTAGE RISE TIME

Mode competition has been calculated using the self-consistent, time dependent, and multimode formalism described in several papers, for example, Refs. 15 and 16. The calculations have been started at 40 kV assuming that at lower voltages no relevant competing modes exist. After each 4 ns, the voltage was increased by 1 kV which corresponds to the beam voltage rise 1 kV/4 ns. Thus, the final operating 65 kV voltage is reached in 100 ns. After this, the calculations have been continued for another 100 ns with a constant voltage 65 kV in order to see whether a stationary state is reached.

The time evolution of amplitudes of competing modes for different magnetic field values is shown in Figs. 1(a)–1(f). At 11.45 T in the stationary state, oscillations in the TE$_{21,2}$ mode are observed. At 11.65 T and 11.70 T, oscillations simultaneously in the TE$_{21,2}$ mode and TE$_{22,2}$ mode are observed, albeit with small amplitudes. At 11.71 T and higher magnetic fields, the gyrotron oscillates in the operating TE$_{22,2}$ mode with a large amplitude and in parasitic modes TE$_{12,5}$ and TE$_{5,8}$ with negligible amplitudes. Frequency mismatches are favorable for all these three modes (see Fig. 3(e)). Mode hopping is observed at 11.45 T during the start-up phase.$^4$ The TE$_{22,2}$ mode is first excited and then replaced with the TE$_{21,2}$ mode.

FIG. 3. Frequency mismatch of the competing modes as a function of time and voltage for different magnetic fields.
In Fig. 2 the output power is shown as a function of time and voltage.

It is obvious that in the case of strong multimoding the output power is low. At the magnetic field 11.72 T, the gyrotron produces about 400 kW output power with the maximum efficiency and at 11.80 T about 330 kW practically only in the operating mode TE$_{22,2}$ for both magnetic fields.

In Figs. 3(a)–3(f), the frequency mismatches for competing modes are shown as functions of time and voltage.

Coupling impedances given in Table I and frequency mismatch curves shown in Fig. 3 allow one to understand the subtle mode competition processes illustrated in Fig. 1. The coupling impedances are large and equal to the TE$_{22,2}$ and TE$_{21,2}$ modes. The coupling impedance of the TE$_{5,8}$ mode is three times smaller, but, at the same time, ten times larger than the coupling of the TE$_{7,7}$ mode. Thus, it is obvious that the competition takes place mainly between three modes: TE$_{22,2}$, TE$_{21,2}$, and TE$_{5,8}$. It is interesting that at $\sim$45 kV, the minimum of the amplitude of the TE$_{22,2}$ mode coincides with the maximum of the amplitude of the TE$_{5,8}$ mode (Figs. 1(c)–1(f)). This can be understood by inspecting the frequency mismatch curves shown in Figs. 3(c)–3(f). At low voltages, the frequency mismatch of the TE$_{5,8}$ mode is much more favorable than for the TE$_{22,2}$ mode. As a result, the TE$_{5,8}$ mode begins to suppress the TE$_{22,2}$ mode. With increasing voltage frequency mismatches continue to improve for both the TE$_{5,8}$ and the TE$_{22,2}$ mode. However, at around 45 kV, the TE$_{22,2}$ mode begins to win the competition due to its much stronger coupling to the electron beam.

![Figure 4](image4.jpg)

**FIG. 4.** Pitch factor as a function of voltage.

![Figure 5](image5.jpg)

**FIG. 5.** Temporal dependence of the field profiles for a fixed magnetic field $B = 11.72$ T.
The variation of the pitch factor during the voltage rise phase is taken into account in the mode competition calculation. In Fig. 4, the dependence of the pitch factor calculated with the analytic equations of $\beta_\perp$ and $\beta_\parallel$ given in Sec. II is plotted as a function of voltage. In a triode gun, the pitch factor depends on the magnetic field. In the considered range of magnetic fields, its average value for the operating voltage 65 kV is 1.2.

In Figs. 5(a)–5(d), the temporal dependence of the axial field profile is shown.

It is interesting to follow the shape of the field profiles with time (voltage). Initially, (a), the field profile of the operating mode has two humps; i.e., its axial index is 2 and this is the backward wave oscillation (BWO) regime. This is consistent with Fig. 3(e) where $\Delta < 0$ up to 12 ns. With increasing time the second hump disappears (transition to gyrotron regime), but the profile remains asymmetric, (b) and (c). At 40 ns (d) the field profile becomes symmetric. With increasing voltage, the stationary state is reached and the field profiles of

![Graphs showing field profiles](image-url)

**FIG. 6.** Absolute values of mode amplitudes at the cavity exit as a function of time and voltage for different magnetic fields. Dotted curves correspond to the TE$_{5,8}$ mode.
parasites $TE_{12,5}$ and $TE_{5,8}$ gradually vanish. It should be noted that the $TE_{21,2}$ mode remains in the backward wave oscillation ($\Delta < 0$) regime throughout the time evolution.

### IV. MODE COMPETITION CALCULATIONS IN THE CASE OF FAST AND INSTANT VOLTAGE RISE TIME

The calculations have been started at 40 kV, assuming that at lower voltages no relevant competing modes exist. After each 0.4 ns, the voltage was increased by 1 kV which corresponds to the beam voltage rise 10 kV/4 ns. Thus, the final operating 65 kV voltage is reached in 10 ns. After this, the calculations have been continued with a constant voltage 65 kV in order to see whether a stationary state is reached. The results of the calculations are shown in Figs. 6(a)–6(f).

At 11.45 T in the stationary state, oscillation in the $TE_{21,2}$ mode is observed. At 11.65 T and 11.70 T, oscillations simultaneously in the $TE_{21,2}$ and $TE_{22,2}$ modes are observed, albeit with small amplitudes. At 11.70 T, stationary state is reached after about 95 ns after a rather long time of re-growing of the $TE_{22,2}$ mode. This re-growing time is almost the same as that in Fig. 1(c). At 11.72 T and 11.80 T, the gyrotron oscillates in the operating $TE_{22,2}$ mode with a large amplitude and in the parasitic modes $TE_{12,5}$ and $TE_{5,8}$ with negligible amplitudes. Frequency mismatches are favorable for all these three modes (see Fig. 3(e)). It is seen that time evolutions of the modes are rather similar to the ones shown in Fig. 1. In particular, the competition between the $TE_{22,2}$ and $TE_{5,8}$ at higher magnetic fields is shown in Figs. 1(c)–1(f). Probably this can be attributed to the fact that the fast voltage rise time is still longer than the cavity filling time $t_{\text{fill}} \sim 1$ ns and the time evolution can be regarded as quasi-static.

In Figs. 7(a) and 7(b), the temporal dependence of the axial field profile is shown. After 2 ns the field profile still has some humps (BWO regime). However already after 4 ns the field profile becomes symmetric. Thus, in the case of the fast voltage rise, the transition to the gyrotron regime occurs earlier than in the case of a slower voltage rise (Fig. 5(b)).

Next we performed calculations with an instant voltage rise time; i.e., it was assumed that at $t = 0$ ns the voltage is already 65 kV to investigate the effects of the voltage rise time faster than the cavity filling time. The voltage of 65 kV was kept throughout the simulation time. The results are shown in Figs. 8(a)–8(f).

Although mode hopping between the $TE_{22,2}$ mode and the $TE_{21,2}$ mode disappears, the stationary oscillation modes are the same independent of the voltage rise time. This is a very favorable feature to realize stable high power oscillation of the $TE_{22,2}$ mode. It is also interesting to observe that the $TE_{5,8}$ mode does not play any role in the competition process. Instead the $TE_{12,5}$ mode tries to survive because its frequency mismatch at 65 kV is more favorable than the frequency mismatch of the $TE_{5,8}$ mode (see Fig. 3(f)).

Albeit the instant voltage rise cannot be realized in experiments, it is useful from the theoretical point of view. As is well known, for the instant voltage rise, the hard-self excitation region cannot be reached without assistance of the competing modes in the lower magnetic field region. Indeed, it can be seen in Figs. 8(c)–8(e) that the operating $TE_{22,2}$ mode needs the assistance of the competing $TE_{21,2}$ mode.

In Figs. 9(a)–9(c), squares of the stationary amplitudes are shown as functions of the magnetic field for the three cases of the voltage rise time. These quantities are proportional to the oscillation power of each mode out of the cavity. Sharp increases in the $TE_{22,2}$ mode power at around 11.70 T are seen and the highest power of the $TE_{22,2}$ mode is attained at 11.72 T for all cases. As shown in Fig. 3, the stationary value of the frequency mismatch is about 0.6.

### V. DISCUSSION AND SUMMARY

The results of the simulation calculations are summarized as follows.

1. Calculations demonstrate that over 300 kW power stable oscillations in the $TE_{22,2}$ mode can be achieved even for the case of the operation with the use of a simple resistive divider. This feature is independent of the voltage rise time, which is partly due the characteristics of WG modes and competition with $\Delta m = -1$ mode can be avoided. Capability of simple operation is advantageous to a diagnostic-use power source.

2. The maximum efficiency of the $TE_{22,2}$ mode is attained at 11.72 T without oscillation of the competing $TE_{21,2}$ mode. In many cases, high power single mode oscillation is realized by slightly increasing the magnetic field strength to avoid competition with lower frequency modes.
modes, which results in a decrease in the efficiency. This is not the case for the present study. The maximum power of the TE$_{22,2}$ mode is realized with the maximum efficiency at 11.72 T regardless of the existence of the TE$_{21,2}$ mode.

(3) In the case of finite voltage rise time longer than the cavity decay time, the transition from the BWO regime (multiple axial peaks) to the gyrotron regime (one axial peak) is observed and is consistent with the temporal dependence of the frequency mismatch.

(4) A slight decrease in the magnetic field to 11.70 T results in the two mode oscillation with a lower power. The TE$_{22,2}$ mode does not sustain in the stationary state without the TE$_{21,2}$ mode because of too large frequency mismatch of about 0.6.

(5) With further decreasing the magnetic field, mode hopping from the TE$_{22,2}$ mode to the TE$_{21,2}$ mode is observed in the start-up phase.

Generally one can say, that start-up scenarios in gyrotrons are very complicated. As is notified in a text book [see page 110 of Ref. 13], in gyrotrons with a very dense spectrum of competing modes even minor effects can change the final equilibrium state, comparison between the results of

FIG. 8. Amplitude as a function of time for different magnetic fields. Voltage is all the time constant 65 kV. Dotted curves correspond to TE$_{5,8}$ mode.
many case studies is important. Transition from the initially excited BWO mode with a multi-peak axial profile to the gyrotron oscillation with a single-peak axial profile was also mentioned in Refs. 7 and 8. Intermittency of the excited high order volume modes was pointed out in Refs. 6 and 7. The present simulation observed mode hopping between $D_{m=1}$ WG modes. An important point of the present study is the attainment of the maximum power of the $TE_{22,2}$ mode with no oscillation of the $TE_{21,2}$ mode. This maximum power is realized without sophisticated start-up scenarios of the cathode voltage or the anode voltage. This is probably due to the mode selection of a WG mode. Another important difference of the present simulation study from the results of Ref. 7 is the lower limit of the voltage rise time. Excitation of a low frequency side competing mode as the stationary mode was predicted for too fast voltage rise time in Ref. 7. Contrary to this, as shown in Fig. 9(b), the $TE_{22,2}$ mode is excited and the highest power oscillation is realized at the steady state even for much faster voltage rise of 2.5 kV/ns. The effect of the voltage overshoot is not calculated because the power supply of the 300 GHz gyrotron shows no overshoot.

The complicated start-up scenario is also observed in other vacuum tubes as recent studies of the influence of voltage rise time on microwave generation in relativistic backward wave oscillator demonstrate. However, the present study shows almost the same evolution of the mode competition for both the slow and fast voltage rise time cases. This is because the cavity decay time is very short compared even with the fast voltage rise time which is extremely fast for the current high voltage switching technology. On the other hand, since the cavity decay time becomes longer for low frequency gyrotrons, evolution of the mode competition may depend on the voltage rise time.

FIG. 9. Squared stationary amplitudes as a function of the magnetic field for different voltage rise times. Circles correspond to the $TE_{21,2}$ mode and triangles to the $TE_{22,2}$ mode.

References: