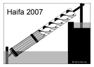
Pumping versus bias:



elements of adiabatic transport theory for recent experiments on double quantum dots

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Total dc current $\mathcal{I} = \mathcal{I}_{\mathrm{pump}} + \mathcal{I}_{\mathrm{bias}} + \mathcal{I}_{\mathrm{mix}}$ consists of

1 $$\mathcal{I}_{\mathrm{pump}} = \frac{e}{h}\iint \frac{f_l'(E)+f_r'(E)}{2}\,\mathrm{Tr}\left[\hat{G}^\dagger\cdot\frac{d}{dt}\mathrm{Re}(\hat{G}^{-1})\cdot\hat{G}\cdot\hat{\Gamma}_l - \mathrm{Re}\,\hat{G}\cdot\frac{d}{dt}\hat{\Gamma}_l\right]dE\,\frac{dt}{\tau}$$

Essentially Brouwer/Buttiker-Pretre-Thomas f-la expressed via equilibrium Green functions [1]

Retarded matrix GF at frozen time t

$\hat{\Gamma}_l + \hat{\Gamma}_r = 2\,\mathrm{Im}(\hat{G}^{-1})$

2 $$\mathcal{I}_{\mathrm{bias}} = \frac{e}{h}\iint [f_l(E)-f_r(E)]\,\mathcal{T}(E)\,dE\,\frac{dt}{\tau}$$

Time average of the Landauer f-la

Transmission coefficient

3 $$\mathcal{I}_{\mathrm{mix}} = \frac{e}{h}\iint \frac{f_r'(E)-f_l'(E)}{2}\,\mathcal{T}(E)\frac{d\varphi_{\mathcal{T}}}{dt}dE\,\frac{dt}{\tau}$$

Additional mixing term [2]

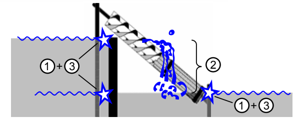
Transmission phase

This general adiabatic current for a locally driven non-interacting quantum system kept at a fixed finite bias was derived in [2].

Same left-to-right current via scattering matrix (TRS)

$$1+3 = \frac{e}{h}\iint\left[-\frac{f_l'+f_r'}{2}\,\mathcal{R}\,\frac{d(\alpha+\varphi_{\mathcal{T}})}{dt} - f_r'\,\mathcal{T}\frac{d\varphi_{\mathcal{T}}}{dt}\right]dE\,\frac{dt}{\tau}$$

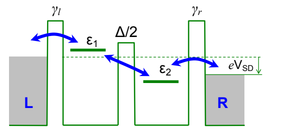
$$\hat{S} = e^{i\varphi_{\mathcal{T}}}\begin{bmatrix}\sqrt{\mathcal{R}}e^{i\alpha} & i\sqrt{\mathcal{T}}\\ i\sqrt{\mathcal{T}} & \sqrt{\mathcal{R}}e^{-i\alpha}\end{bmatrix}$$



- No bias (pumping only): 2+3 vanishes, 1 depends on R and α only
- Very large bias: only 2 remains if all resonances stay in bias window
- Intermediate: all three terms are important

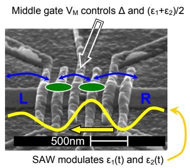
Here we explore 1 and 2 for a specific model, but keep 3 in mind.

A model for a double quantum dot



$$\mathcal{H} = \sum_{i=1,2}\epsilon_i d_i^\dagger d_i + (d_1^\dagger d_2 + d_2^\dagger d_1)(\Delta/2) + \sum_{k;i=1,2}\epsilon_{ki}c_{ki}^\dagger c_{ki} + \sum_{k;i=1,2}\left(V_{ki}c_{ki}^\dagger d_i + \mathrm{h.c.}\right)$$

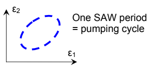
Middle gate V_M controls Δ and $(\varepsilon_1+\varepsilon_2)/2$



SAW modulates $\varepsilon_1(t)$ and $\varepsilon_2(t)$

Pumping by Surface Acoustic Waves (SAW)

$$\begin{cases}\epsilon_1(t) &= \epsilon_0 - \Delta E/2 + P\cos(\omega t - \varphi/2)\\ \epsilon_2(t) &= \epsilon_0 + \Delta E/2 + P\cos(\omega t + \varphi/2)\end{cases}$$



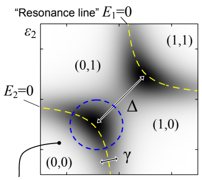
One SAW period = pumping cycle

φ is set by SAW wavelength / interdot distance

$\tau^{-1} \equiv \omega/(2\pi) = 2 \div 5\ \mathrm{GHz}$

2 Theory

"Resonance line" E_1=0



2D plot of $\mathcal{T}(E=0)$

Eigenstates of an isolated dot (γ=0):

$$E_{1,2} = (\epsilon_1+\epsilon_2 \pm \sqrt{(\epsilon_1-\epsilon_2)^2+\Delta^2})/2$$

Instantaneous transmission:

$$\mathcal{T}(E) = \frac{4\Delta^2\gamma^2}{(4[E-E_1]^2+\gamma^2)(4[E-E_2]^2+\gamma^2)}$$

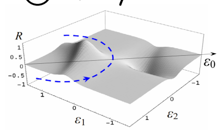
Double peak for Δ » γ ("two triple points")

Single peak for Δ « γ ("one quadruple point")

Linear conductance with SAW on is just an average along the "pumping contour" !

$$2 \Rightarrow G = \frac{e^2}{h}\oint \mathcal{T}(\epsilon_1,\epsilon_2)\,\frac{dt}{\tau}$$

1 Theory



For any contour,
the pumping current is an area integral:

$$\mathcal{I}_{\mathrm{pump}} = \oiint R(\epsilon_1,\epsilon_2)\,d\epsilon_1\,d\epsilon_2$$

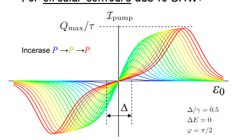
where R is available analytically

$$R(\epsilon_1,\epsilon_2) = \frac{32\gamma^3\Delta^2(\epsilon_1+\epsilon_2)}{\pi\left\{\gamma^4+2\left[\Delta^2+2\left(\epsilon_1^2+\epsilon_2^2\right)\right]\gamma^2+\left(\Delta^2-4\epsilon_1\epsilon_2\right)^2\right\}^2}$$

$$Q_{\max}/e = 1-\left(\frac{\gamma}{\sqrt{\gamma^2+\Delta^2}}\right)^3 = \begin{cases}1-(\gamma/\Delta)^3 & \gamma\ll\Delta\\ 3\Delta^2/(2\gamma^2) & \gamma\gg\Delta\end{cases}$$

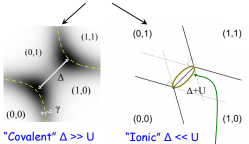
Pumped charge quantization [1,2] if large contour around "triple point"

For circular contours due to SAW:



- Current sign reversal as ε_0 goes across a single conductance peak ("quadruple point", Δ < γ)
- "+" and "-" peaks:
  - ✓ grow linearly with P at P << Δ
  - ✓ saturate at P ≈ Δ,
  - ✓ move apart at P >> Δ

Double quantum dots: role of interactions

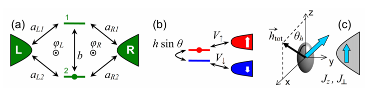


"Covalent" Δ >> U   "Ionic" Δ << U

- Intra-dot Coulomb interaction is crudely approximated by spinless
- Inter-dot $U_{n_1 n_2}$ can vary!
- Our U=0 theory cover "covalent" only

Good news [7,8]: if then ground state is a Fermi liquid, then 1 works !

In the gap of single occupancy one can get very accurate zero temperature Green function for Δ, ΔE , γ « U from the phase shifts, see [9].



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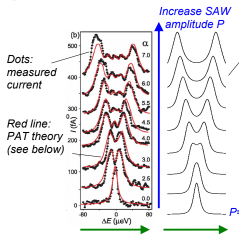
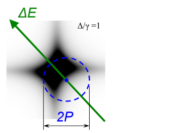
2 Theory vs experiment

Naber et al. experiment [3]:

1. Fix bias
2. Scan ΔE with gates
3. Measure dc current at various SAW powers



Tune with gates $\Delta E \equiv \epsilon_2 - \epsilon_1$



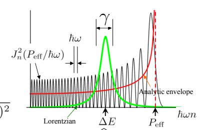
Results are not sensitive to bias value: increasing V_{SD} means projecting more and more of the 2D plot onto ΔE axis

2 More theory

Non-adiabatic theory of Stoof & Nazarov

For large bias, finite ω and Δ<<γ,
photo-assisted-tunneling (PAT) result [4]:

$$\mathcal{I}^{\mathrm{PAT}} = \frac{2\pi e\Delta^2\gamma_r}{h}\sum_{n=-\infty}^{+\infty}\frac{J_n^2(P_{\mathrm{eff}}/\hbar\omega)}{\gamma_r^2/4+(\Delta E-n\hbar\omega)^2}$$



Here $\epsilon_2(t)-\epsilon_1(t) \equiv \Delta E + P_{\mathrm{eff}}\sin\omega t \Rightarrow P_{\mathrm{eff}} = 2P\sin(\varphi/2)$

Adiabatic limit $\mathcal{I}^{\mathrm{PAT}}_{\omega\to 0} = \frac{e}{\hbar}\int\frac{\Delta^2\gamma_r}{\gamma_r^2/4+(\epsilon_1-\epsilon_2)^2}\frac{dt}{\tau}$ is accurate when $\boxed{\hbar\omega \leq \gamma}$!

Confirmation within the present formalism:

$$\mathcal{I}_{\mathrm{bias}}(t) = \frac{e}{h}\int\mathcal{T}dE = \frac{e}{\hbar}\frac{\Delta^2(\gamma_l+\gamma_r)}{(\gamma_l+\gamma_r)^2(1+\Delta^2/\gamma_l\gamma_r)+4(\epsilon_1-\epsilon_2)^2}$$

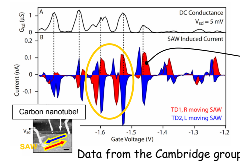
(For $eV_{\mathrm{SD}} \gg \gamma, |\epsilon_{1,2}|$)

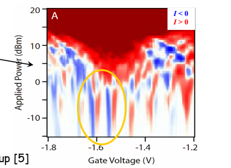
Under conditions of [4], $\Delta \ll \gamma_l \ll \gamma_r \Rightarrow \boxed{\mathcal{I}_{\mathrm{bias}} = \mathcal{I}^{\mathrm{PAT}}_{\omega\to 0}/4}$

Time-averaged Landauer for spinless, non-interacting electrons

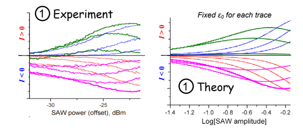
Adiabatic limit from reduced density matrix solution, spinless electrons with stong interdot repulsion, eV_{SD}<<U→∞.

1 Experiment





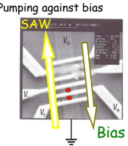
Data from the Cambridge group [5]



Preliminary calculation (VK, unpublished) with the present model

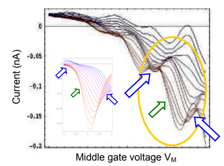
1 + 2 Exp & theory (work in prgress)

Gate-defined double dot with Δ « γ
Pumping against bias



Preliminary experimental data
by Berd Kästner (NPL, UK)

Ongoing improvements:
- ✓ separate gate to keep Δ=const more accurately
- ✓ triple to quadruple point crossover
- ✓ full bias scan from 0 and up
- ✓ ultimate goal – resolve $\hbar\omega$



- SAW degrades conductance at the peak, but promotes at wings ⇒ Bunching
- One side gets "+" from the pumping part, the other part gets "-" ⇒ enhanced slope