

# Anderson localization: Metallic behavior in two dimensions?

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# 1. Definitions

## 1.1. From Fermat's last theorem to Anderson theorem

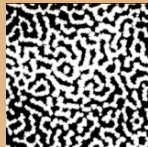
### 1.1.1. Fermat's last theorem

The equation

$$x^D + y^D = z^D$$

has **no** non-zero integer solutions for  $x$ ,  $y$  and  $z$  when  $D > 2$ .

- Solutions for  $D = 1$  (trivial).
- Solutions for  $D = 2$  (Pythagoras' Theorem).
- No solutions for  $D > 2$  (Fermat, Euler, Cauchy, Lamé, Kummer, . . . , Wiles-1993)



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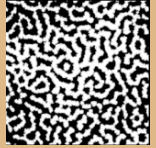
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### 1.1.2. Phase transition in magnetics: Ising model

The Ising model undergoes a phase transition between an ordered and a disordered phase in  $D \geq 2$ .

- Exact solution for  $D = 1$  (Ising) - **no phase transition.**
- Exact solution for  $D = 2$  (Onsager) - **second order phase transition.**



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### 1.1.3. Phase transition in magnetics: Heisenberg model

No exact solutions for 2D!

Condensed-Matter Physics (1986)

We believe that the 2-D Heisenberg model has no phase transition at all - it remains paramagnetic (disordered) at all temperatures other than zero.

H. E. Stanley and T. A. Kaplan, PRL, 17, 913 (1966)

Abstract:

We point out that the existence of a phase transition - as indicated by extrapolation from high-temperature expansions - is as well-founded for two-dimensional lattices with nearest-neighbor ferromagnetic Heisenberg interactions as for three-dimensional lattices, and that the "well-known result" that there exists no phase transition in two dimensions is not a valid conclusion from the standard spin-wave argument.



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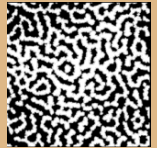
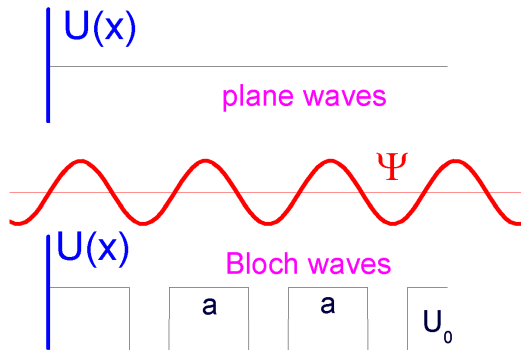
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## 1.2. Ideal crystal

- The periodicity of the crystal
- Classification of electronic wave functions as Bloch waves
- Calculation of band structures



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### 1.3. Extended state

In real life the crystalline state is the exception rather than the rule.

#### Disorder:

- **Weak-disorder limit** (few impurities or interstitials in an otherwise perfect crystalline)
- **Strongly disordered limit** (alloys or glassy structures).

The weak-disorder limit is traditionally described by the **scattering** of Bloch waves by impurities.



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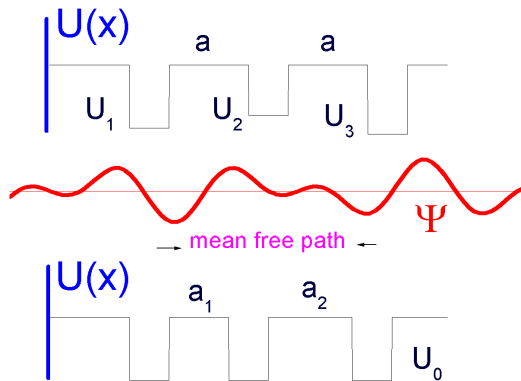
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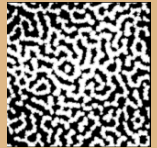
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The traditional view had been that scattering by the random potential causes the Bloch waves to lose phase coherence on the length scale of the **mean free path**. The wave function remains **extended** throughout the sample.



Typical wave functions of extended state with mean free path



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P.W. Anderson, Phys.Rev. 109, 1492 (1958)

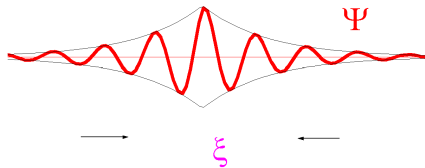
If the disorder is very strong, the wave function may become localized: the envelope of the wave function decays exponentially,

$$\Psi(r) \propto \exp(-r/\xi),$$

and  $\xi$  is the **localization length**.

- Extended state = **metallic state**
- Localized state = **insulating state**

Transition between a metal and an insulator = **metal-insulator transition** (MIT)



Typical wave functions of localized state with localization length  $\xi$



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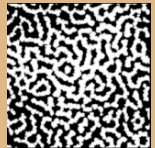
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## Philip Warren Anderson - Autobiography



The years since the Nobel Prize (1977) have been productive ones for me. For instance, in 1978, shortly after receiving the prize in part for localization theory, I was one of the "Gang of Four" (with Elihu Abrahams, T.V. Ramakrishnan, and Don Licciardello) who revitalized that theory by developing a scaling theory which made it into a quantitative experimental science with precise predictions as a function of magnetic field, interactions, dimensionality, etc.; a major branch of science continues to flow from the consequences of this work.



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#### 1.4. Theoretical consensuns: no metallic behavior in two dimensions!

- All states are localized in one (1-D) dimensions.
- All states are localized in **two**<sup>†</sup> (2-D) dimensions.
- MIT is a **continuous**<sup>†</sup> one (second-order) in three (3-D) dimensions.

<sup>†</sup>Scaling theory of localization:

Abrahams, Anderson, Licciardello and Ramakrishnan,  
Phys.Rev.Lett. 42, 673 (1979)



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## 1.5. L. Onsager and R. Feynman

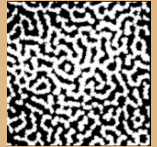
Then Professor Onsager got up and said in a dour voice, “Well, Professor Feynman is new in our field, and I think he needs to be educated. There’s something he ought to know, and we should tell him.”

I thought, “Geesus! What did I do wrong?”

Onsager said, “We should tell Feynman that *nobody* has ever figured out the order of *any* transition correctly from first principles ...”

Richard P. Feynman

”Surely You’re Joking, Mr Feynman!”



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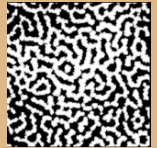
## 1.6. Experiment

### Colloquium: Metallic behavior and related phenomena in two dimensions

E. Abrahams, S.V.Kravchenko and M.P.Sarachik,  
*Reviews of Modern Physics*, 73, 251 (2001)

#### Abstract:

For about 20 years, it has been the prevailing view that **there can be no metallic state or metal-insulator transition in two-dimensions** in zero magnetic field. In the last several years, however, unusual behavior suggestive of such a transition has been reported in a variety of dilute two dimensional electron and hole systems. The physics behind these observations is at present not understood. The authors review and discuss the main experimental findings and suggested theoretical models.



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## 2. Introduction

### 2.1. IOP

INSTITUTE OF PHYSICS PUBLISHING

JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 13777–13797

PII: S0953-8984(02)53297-5

# Exact analytic solution for the generalized Lyapunov exponent of the two-dimensional Anderson localization

V N Kuzovkov<sup>1,2</sup>, W von Niessen<sup>2</sup>, V Kashcheyevs<sup>1</sup> and O Hein<sup>2</sup>

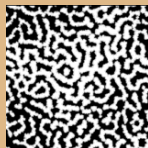
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## Referee A:

This is a most unusual paper, for in it the authors claim to provide an analytical solution to one of the major problems in condensed matter theory: two-dimensional Anderson localization in a disordered tight binding model.

The conventional scaling-based view of  $D=2$  is that all states are localized for arbitrarily weak disorder.

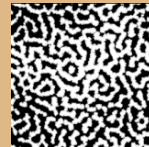
In contrast the present authors conclude that both extended and localized states may arise, and indeed may coexist in such a way that the localized states are not rendered extended (phase coexistence).

This is a radical departure from the conventional view.

- If the present work is correct, it is highly important.
- Is it correct? I can only be honest and say I do not know.
- But if the work is correct, it will become a classic.

## Referee B:

- If the conclusions of the paper are correct then much of our understanding of this problem will have to be revised.



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## 2.2. PRL

### Exact analytic solution of the 2-dimensional Anderson localization

V.N. Kuzovkov,<sup>1,2</sup> V. Kashcheyevs,<sup>1</sup> O. Hein,<sup>2</sup> and W. von Niessen<sup>2</sup>

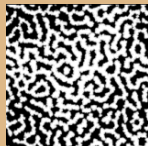
<sup>1</sup>*Institute of Solid State Physics, University of Latvia, 8 Kengaraga Street, LV - 1063 RIGA, Latvia\**

<sup>2</sup>*Institut für Physikalische und Theoretische Chemie, Technische Universität Braunschweig,  
Hans-Sommer-Straße 10, 38106 Braunschweig, Germany*

(Dated: 2001)

The Anderson localization problem in two dimensions is solved analytically via calculation of the Lyapunov exponents. For certain energies and small disorder extended and localized states coexist. The phase of delocalized states is marginally stable. We demonstrate that the metal-insulator transition should be interpreted as a first-order phase transition.

PACS numbers: 72.15.Rn, 71.80.+h



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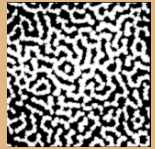
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- **Referee A:** The authors claim to have an exact calculation of the localization length in the two-dimensional Anderson model. If that were true, it would be an exciting result.
- **Referee B:** This paper proposes an ‘exact analytic’ solution of the 2-d Anderson localization problem. If this was true, it would be a major breakthrough.
- **Referee C:** This paper deals with a very important topic, and the results of the paper are sensational. The authors claim to solve exactly the longstanding problem of the two-dimensional localization with highly unexpected results. Namely, they claim that Anderson transition in 2-d is of the first order, and that the localized and conducting states can co-exist. If the results were correct, they would constitute a breakthrough compared only probably with Anderson’s original paper.



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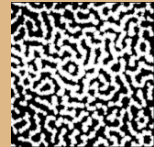
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- **Referee A:** In short, neither the model addressed, nor the solution presented can apply to the problem of Anderson localization in two dimensions. I cannot recommend publication.
- **Referee C:** Concluding, it is my option, that the manuscript has nothing to do with the problem of 2-d localization and, therefore, should be rejected.
- **Referee D:** I agree with the criticism expressed by all three referees A,B,C. The best one can say about this paper is that it possibly considers some other model. This paper should not be published in PRL.

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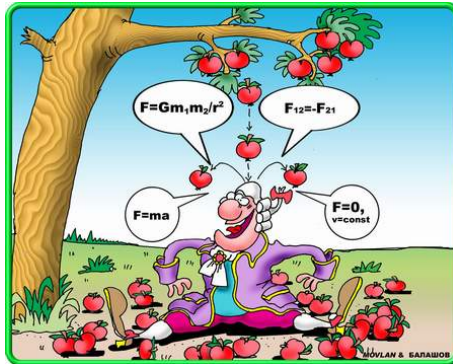
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## Referee D:

- It is not the job of the referees to find out what this model might be and how it relates to the canonical localization problem. Rather, it is the responsibility of the authors to explain why they get a **different result** from what is **accepted** and why their result should be the correct one.



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- Although there is **no general analytical solution** available, there is **consensus** that in two-dimensions all states are localized.
- The fact that certain **experimental observations in Si-MOSFET's and other systems** appear to suggest a metallic phase in a disordered, interacting 2-D electron system **should not be used to argue that we do not understand disordered systems of noninteracting electrons.**
- This is because if there is indeed a phase transition in these systems (which remains to be proven beyond doubt) it is most likely related to the **Coulomb interaction** or possibly the **spin-orbit interaction.**
- For weak disorder the localization length ... can be calculated in the **perturbation theory...** (Results) are confirmed by **numerical studies...**



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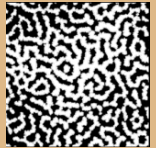
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## 2.3. Paper



- Therefore the **metal-insulator transition should be looked at from the basis of first-order phase transition theory**. This opinion differs from the traditional point of view, which considers this transition as continuous (second-order).
- We have shown that in principle there is the **possibility for the phase of delocalized states in a 2-D disordered non-interacting electron system**. This phase has its proper existence region, but it belongs to the marginal type of stability.



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### 3. New concept 1-D

#### 3.1. Tight-binding equation

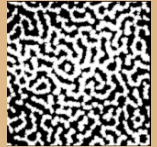
#### Standard form for 1-D:

$$\psi_{n+1} + \psi_{n-1} = (E - \varepsilon_n)\psi_n.$$

The lattice constant and the hopping matrix element are equal to unity.

The on-site potentials  $\varepsilon_n$  are independently and identically distributed with existing first two moments,  $\langle \varepsilon_n \rangle = 0$  and  $\langle \varepsilon_n^2 \rangle = \sigma^2$ .

$\sigma$  - parameter of disorder.



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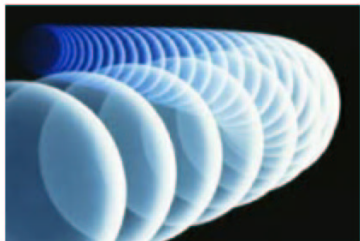
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## 3.2. Recursion relation

Recursion relation and **time evolution** of 1-D system



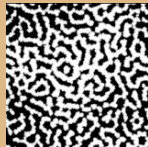
**Semi-infinite system**,  $n \geq 0$

Recursion relation ( $n = 1, 2, \dots$ ):

$$\psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}.$$

$$m \frac{d^2 x}{dt^2} = F(x, t).$$

It turns out to be convenient to consider the index  $n$  not as a **spatial coordinate**, but as **discrete time**. Then equation describes the time evolution of a  $D-1 = 0$  dimensional system.



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### 3.3. Causality in the tight-binding equation



#### Initial condition:

The equation

$$\psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}$$

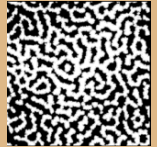
is solved with an initial condition

$$\psi_0 = 0, \psi_1 = \alpha.$$

#### Causality:

In a formal solution of this recursion relation the amplitude  $\psi_{n+1}$  depends only on the random variables  $\varepsilon_{n'}$  with  $n' \leq n$ .

Grid amplitudes on the r.h.s. are **statistically independent** with respect to  $\varepsilon_n$ .



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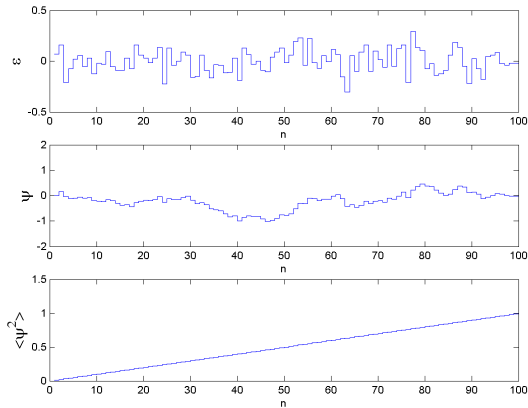
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### 3.4. Additive noise: Normal diffusion

## The equation for random walk

$$\psi_{n+1} = \psi_n + \varepsilon_n$$



Bounded motion,  $\psi_n \equiv \psi_0 = 0$ , for  $\sigma = 0$ .  
Divergence of the second moment for  $\sigma > 0$ :

$$\langle \psi_n^2 \rangle = \sigma^2 n.$$



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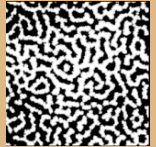
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### 3.5. Multiplicative noise: Generalized diffusion

$$\psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}$$



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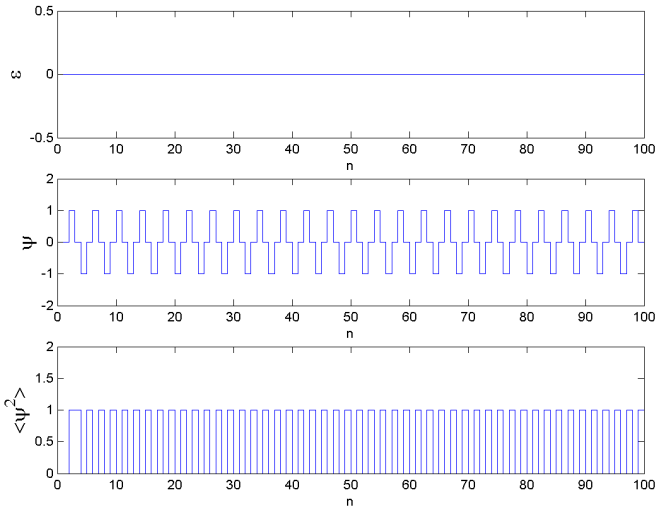
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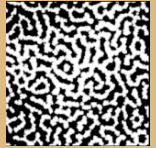
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Bounded motion (proper dynamics),  $[\psi_n] < \infty$ , for  $\sigma = 0$ .



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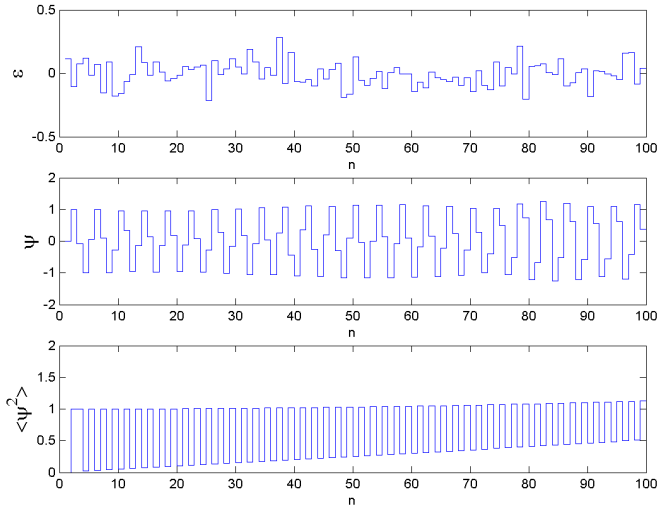
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**Divergence of the second moment for  $\sigma > 0$ :**

$\langle \psi_n^2 \rangle = f(n)$  with  $\lim_{n \rightarrow \infty} f(n) = \infty$ .

### 3.6. Lyapunov exponents $\gamma$ and order parameter

#### Lyapunov exponent

Asymptotic for  $n \rightarrow \infty$ :  $f(n) \propto \exp(2\gamma n)$  .

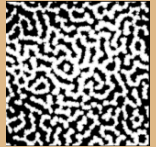
$\psi^2$ -definition:

$$\gamma = \lim_{n \rightarrow \infty} \frac{1}{2n} \ln(\langle [\psi_n]^2 \rangle).$$

**Order parameter:**

$\gamma \equiv 0$  - **Extended states** (no generalized diffusion)

$\gamma \neq 0$  - **Localized states** (generalized diffusion)



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### 3.7. Equations for second moments

We are interested in the **mean behavior** of  $\psi_n^2$ .

**Definitions:**

$$x_n = \langle \psi_n^2 \rangle$$

and

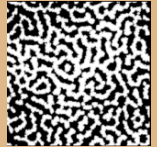
$$y_n = \langle \psi_n \psi_{n-1} \rangle.$$

The basic equation for the variable  $x_n$  is obtained by squaring both sides of recursion relation and averaging over the ensemble.

$$\begin{aligned}x_{n+1} &= (E^2 + \sigma^2) x_n - 2E y_n + x_{n-1}, \\ y_n &= E x_n - y_{n-1}.\end{aligned}$$

The initial conditions are:

$$x_0 = 0, x_1 = \alpha^2, y_0 = 0.$$



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### 3.8. Transfer matrix

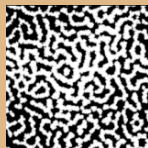
The equations can be rewritten in the form

$$w_{n+1} = T w_n ,$$

where the vector  $w_n$  and the transfer matrix  $T$  are respectively

$$w_n = \begin{pmatrix} x_{n+1} \\ x_n \\ y_{n+1} \end{pmatrix} , T = \begin{pmatrix} E^2 + \sigma^2 & 1 & -2E \\ 1 & 0 & 0 \\ E & 0 & -1 \end{pmatrix} .$$

Initial conditions transform into  $w_0 = \{\alpha^2, 0, 0\}$ .



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### 3.9. Characteristic equation

An explicit formula for  $w_n$  is derived by diagonalizing the transfer matrix  $T$ . The **characteristic equation** of the **eigenvalue problem** for the  $T$  matrix is

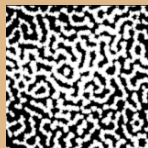
$$\mathcal{D}(\lambda) = 0$$

with function

$$\mathcal{D}(z) = z^3 - (E^2 + \sigma^2 - 1)z^2 + (E^2 - \sigma^2 - 1)z - 1.$$

The resulting exact formula for  $x_n$  is

$$\begin{aligned} x_n/\alpha^2 &= \frac{\lambda_1^n(1 + \lambda_1)}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)} + \\ &+ \frac{\lambda_2^n(1 + \lambda_2)}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)} + \\ &+ \frac{\lambda_3^n(1 + \lambda_3)}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}. \end{aligned}$$



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### 3.10. Z-transform

An alternative solution makes use of the so-called **Z-transform**.

$$X(z) = \sum_{n=0}^{\infty} \frac{x_n}{z^n}, Y(z) = \sum_{n=0}^{\infty} \frac{y_n}{z^n}.$$

The inverse **Z-transform** is quite generally defined via contour integrals in the complex plane

$$x_n = \frac{1}{2\pi i} \oint X(z) z^n \frac{dz}{z}.$$

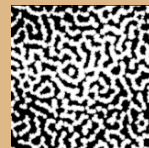
**Properties:**

for the **Z-transform**

$$x_{n+n_0} \Rightarrow z^{n_0} X(z),$$

for the inverse **Z-transform**

$$\frac{z}{z - \lambda} \Rightarrow \lambda^n.$$



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## Determination of the poles

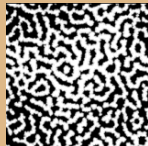
$$\begin{aligned}zX &= (E^2 + \sigma^2)X - 2EY + z^{-1}X + \alpha^2 \\ Y &= Ez^{-1}X - z^{-1}Y\end{aligned}$$

This is easily solved for  $X(z)$

$$X(z) = \frac{\alpha^2 z(z+1)}{\mathcal{D}(z)}$$

where the function  $\mathcal{D}(z)$  is defined above.

- the **solution of the characteristic equation**  $\mathcal{D}(\lambda) = 0$  for the transfer matrix is **equivalent to the determination of the poles** of the  $X(z)$  function,  $\mathcal{D}(z) = 0$ .
- The eigenvalues  $\lambda_i$ , i.e. the poles, determine the asymptotic behavior of the solution  $x_n$  for  $n \rightarrow \infty$ .



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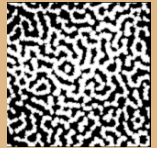
## 4. New concept 2-D

### 4.1. Recursion relation

#### Causality in the tight-binding equation for 2-D

2-D lattice with **one boundary**. The layers of this system are enumerated by an index  $n = 0, 1, \dots$  starting from the boundary, index  $m \in (-\infty, +\infty)$ . Schrödinger equation in the form of a recursion relation:

$$\psi_{n+1,m} = (E - \varepsilon_{n,m})\psi_{n,m} - \psi_{n-1,m} - \sum_{\mu=\pm 1} \psi_{n,m+\mu}.$$



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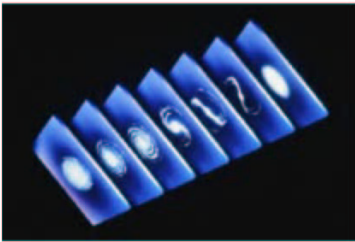
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## Initial condition

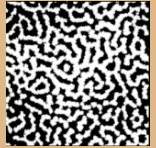
$$\psi_{0,m} = 0, \psi_{1,m} = \alpha_m.$$

Recursion relation describes the **time evolution** of a  $D - 1 = 1$  dimensional system.

**Causality:** Grid amplitudes on the r.h.s. are **statistically independent** with respect to  $\varepsilon_{n,m}$ .



Recursion relation and **time evolution** of 2-D system



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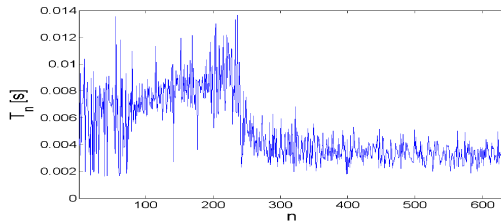
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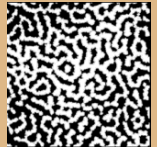
## 4.2. Concept of filters

T.F.Weiss: Signals and systems. Lecture notes.  
<http://umech.mit.edu/weiss/lectures.html>

- **Signals** are variables that carry information.



- **Systems** process **input signals** to produce **output signals**.



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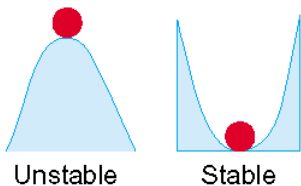
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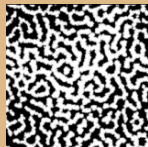
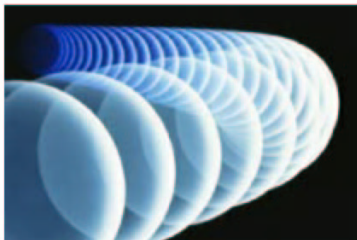
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- A system is **BIBO** stable if every **Bounded Input** leads to a **Bounded Output**.



- Physical systems with **time** as independent variable are **causal** systems.



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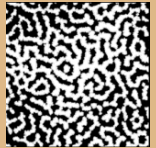
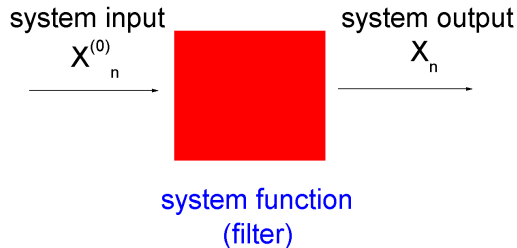
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## Linear and causal filter

- Input signal  $x_n^{(0)}$
- Output signal  $x_n$
- **Linear** filter  $h_n$
- **Causal** filter ( $h_n = 0$  for  $n < 0$ )

### SIGNAL THEORY



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- Convolution property

$$x_n = \sum_{l=0}^n h_{n-l} x_l^{(0)}.$$

- Z-transform

$$X(z) = H(z)X^{(0)}(z).$$

## Stable and unstable filter

- **Stable filter:** Bounded output for bounded input
- **Unstable filter:** Unbounded output for bounded input

### Unstable filter:

$$\lim_{n \rightarrow \infty} h_n \rightarrow \infty$$



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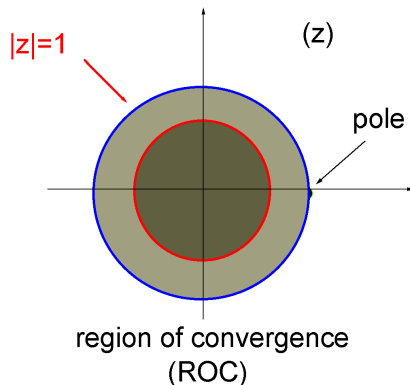
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### 4.3. Signal theory

- The **causal filters** have always **ROC**s (**region of convergence**) outside a circle that intersects the pole with  $\max[\lambda_i]$ .
- causal filter is **stable** (bounded input yields a bounded output) if the **unit circle**  $|z| = 1$  is in the **ROC**.



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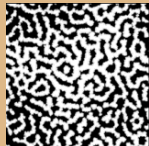
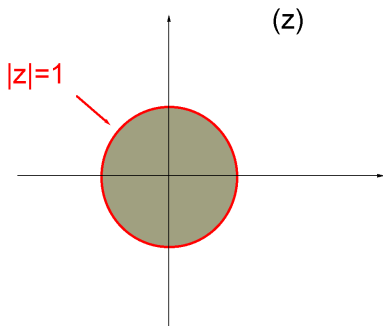
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## For 2-D Anderson localization problem:

$$\frac{1}{H(z)} = 1 - \frac{\sigma^2(z+1)}{2\pi(z-1)} \int_{-\pi}^{\pi} \frac{dk}{w^2 - \mathcal{L}^2(k)},$$
$$\mathcal{L}(k) = E - 2 \cos(k),$$
$$w^2 = \frac{(z+1)^2}{z}.$$



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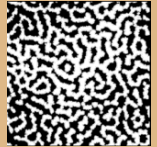
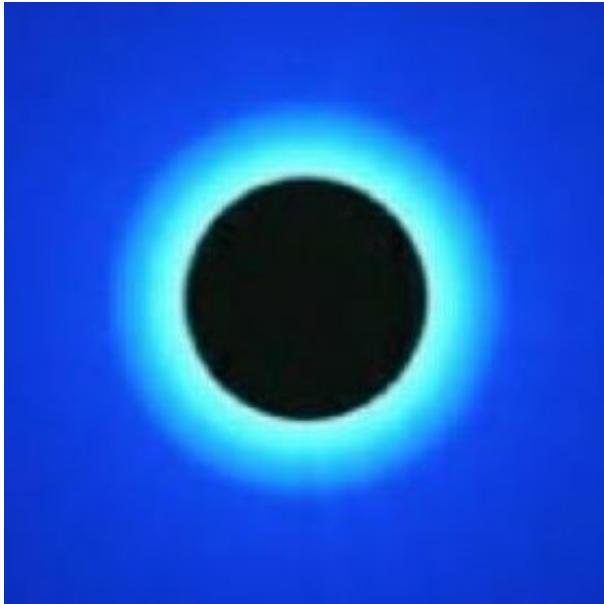
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## 4.4. Two phases solution



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**Unstable filter**  $H_+(z)$ ,  $|z| \geq 1$   
**Band center** ( $E = 0$ )

- Filter

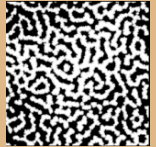
$$H_+^{-1}(z) = 1 - \frac{\sigma^2 z}{(z - 1)^2}.$$

- Inverse Z-transform

$$h_n^+ = \delta_{n,0} + 2 \tanh(\gamma) \sinh(2\gamma n)$$

(exponentially growing function,  $2 \sinh(\gamma) = \sigma$ ).

- The localization length is  $\xi = \gamma^{-1}$ .
- Interpretation - **localized states**.



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**Stable filter**  $H_-(z)$ ,  $|z| \leq 1$   
**Band center** ( $E = 0$ )

- Filter

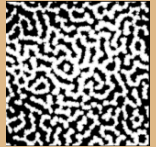
$$H_-^{-1}(z) = 1 + \frac{\sigma^2 z}{(z - 1)^2}.$$

- Inverse Z-transform

$$h_n^- = \delta_{n,0} + 2 \tan(\phi) \sin(2\phi n)$$

(bounded oscillating function,  $2 \sin(\phi) = \sigma$ ).

- Marginal stability.
- The filter exists only for **small disorder** ( $\sigma < 2$ ).
- Interpretation - **delocalized states**.



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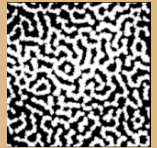
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## 4.5. Coexistence of phases

- The solution  $H(z)$  may be interpreted as representing **two phases**, considering that  $H_+(z)$  determines the properties of the insulating state, but  $H_-(z)$  — the metallic state. Therefore the **metal-insulator transition** should be looked at from the basis of **first-order phase transition theory**.
- A macroscopic system for first-order phase transition is heterogeneous and consists of macroscopically homogeneous domains of the two phases.
- The **self-averaging** quantities tend always to be **unphysical** because the **average over an ensemble** always includes an **average over the phases**.



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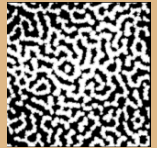
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Physically meaningful in this case are only the properties of the pure phases.

The trivial example in this respect is the **coexistence of water and ice**. An average density of this system has no sense and depends on the relative fraction of the phases, whereas the density of **pure water** and **pure ice** are meaningful quantities.



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## Microscopic structure of the metal-insulator transition in two dimensions

S.Ilani, A.Yacoby, D.Mahalu and H.Shtrikman,  
*Science*, 292, 1354 (2001)

### Abstract:

A single electron transistor is used as a local electrostatic probe to study the underlying spatial structure of the metal-insulator transition in two-dimensions. The measurement show that as we approach the transition from the metallic side, a **new phase** emerges that consists of **weakly coupled fragments** of the two-dimensional system. **These fragments** consist of localized charge that **coexists** with the surrounding **metallic phase**. As the density is lowered into the insulating phase, the number of fragments increases on account of the disappearing metallic phase.



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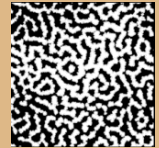
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Vladimir Kuzovkov and Wolfgang von Niessen



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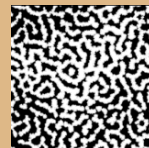
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Slava Kashcheyevs



Oliver Hein



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