Anderson localization: surprise from theory and experiment

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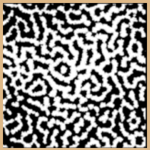
February 22, 2010



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1. Introduction

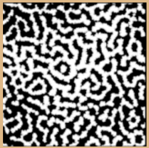
1.1. Laboratory of Theoretical Physics and Computer Modeling

http://www.cfi.lu.lv/teor

- Dr.hab. E.Kotomin, head of laboratory
- Dr.hab. V.Kuzovkov, deputy head of laboratory

In the last 5 years:

With 11 people in our laboratory (less than 5% of scientists in Latvia working in materials science area) we published 140 papers, i.e. 27% of publications and constitute 27% of junior scientists received PhD.



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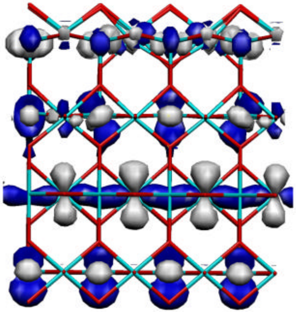
The participation in 5 International projects in the field of nano-materials and ecologically clean new energy sources.

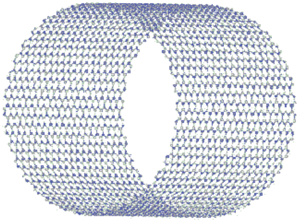
- **CFI: Euratom-Latvia Fusion project, where new materials for the first-reactor-wall exposed to an intensive radiation and high temperatures (e.g. in Tokamak-type thermo-nuclear reactors) are modeled;**
- **CFI: ERA-Net MATERA project dealing with a new generation of non-volatile high capacity memories;**
- **Laboratory: FP7 "Catherine" project focused on development of advanced nano-tubes, nano-connects and their architectures for nano-electronics;**
- **Laboratory: FP7 "F-bridge" project dealing with development of new nuclear fuels for fission-type (atomic) reactors of the new generation-IV;**
- **Laboratory: FP7 "NASA-OTM" project focused on the development of new ceramic membranes for gas separation and CO_2 trapping in gas exhaust of the fossil fuel plants (Zero-CO_2 emission International activity).**

1.2. Research interests

1.2.1. Atomic/electronic structure calculations of the defects in solids

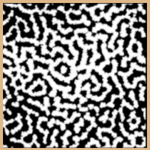
(E.Kotomin, Y.Zhukovskii,. . .)
The Laboratory is a leader in the Baltic region in the large-scale first-principles modeling of the atomic and electronic structure of technologically important materials with a focus of the nano-materials, defects, surfaces and interfaces.



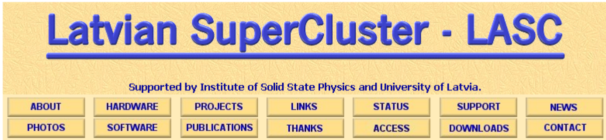


Recent books:

- **E.A. Kotomin, K. Sickafus, Radiation Effects in Solids, NATO School Proceedings, Kluwer, 2006;**
- **E.A. Kotomin and C.R.A. Catlow, Computer Modeling of Materials, Kluwer, 2001.**



1.2.2. Latvian SuperCluster - LASC

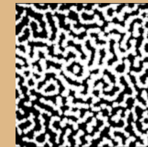


Linux Powered

Powered by APACHE

LASC is a Beowulf-like Linux cluster.
The total resources available to the users are:

- **126 CPUs cores;**
- **the theoretical peak performance of about 256.2 GFLOPS;**
- **252 GB RAM memory;**
- **15.4 TB total hard disk space.**



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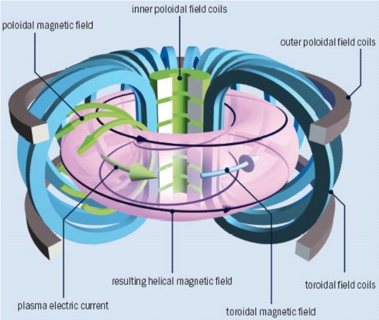
1.2.3. Euroatom

(O.Dumbrajs)

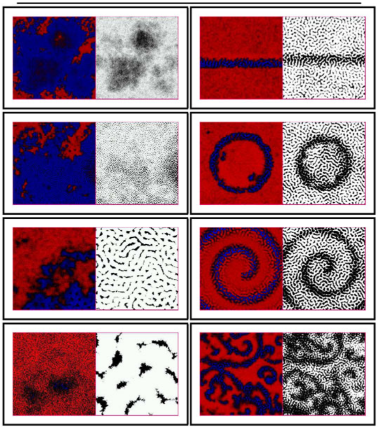
- Stochastization of magnetic fields and magnetic reconnection;
- Development of gyrotrons.



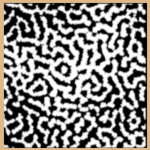
Our research is performed in cooperation with many partners in the framework of a number of integrated programmes, e.g., EUROATOM (both fission [modelling of advanced nuclear fuels, E.Kotomin] and fusion [Y-strengthened radiation-resistant steels, Y.Zhukovskii])

1.2.4. Cooperative effects in the kinetics of bimolecular reactions

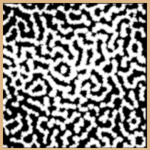
(V.Kuzovkov, E.Kotomin, . . .)



The study of many-particle effects in the kinetics of bimolecular reactions in condensed matter, including radiation defects. Surface-induced reactions are known to play a very important role in heterogeneous catalysis. We study these reactions with emphasis on such fundamental phenomena as pattern formation, reactant self-organization, regular and irregular reactant concentration oscillations as well as chaotic behavior in the case of simple reactions on low-index crystalline surfaces.

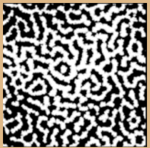


- Kotomin E.A. and Kuzovkov V.N., *Modern Aspects of Diffusion-Controlled Processes: Cooperative Phenomena in Bimolecular Reactions.* - Amsterdam: Elsevier (Vol. 34 in a series of Comprehensive Chemical Kinetics), 1996, 612 p.
- Kuzovkov V.N. and Kotomin E.A., *Kinetics of Bimolecular Reactions in Condensed Media.* - Rept. Progr. Phys., 1988, 51, p. 1479-1524.
- Kotomin E.A. and Kuzovkov V.N. , *Phenomenological Theory of the Recombination and Accumulation Kinetics of Radiation Defects in Ionic Solids.* - Rept. Progr. Phys., 1992, 55, p. 2079-2202.
- Schnörer H., Kuzovkov V.N. and Blumen A. Phys.Rev.Lett. 1989, 63, p.805-808.
- Kuzovkov V.N. and Kotomin E.A. Phys.Rev.Lett. 1994, 72, p.2105-2108.
- Kortlüke O., Kuzovkov V.N. and von Niessen W. Phys.Rev.Lett. 1998, 81, p.2164-2167.
- Kuzovkov V.N., Kortlüke O. and von Niessen W. Phys.Rev.Lett. 1999, 83, p.1636-1639.
- Kortlüke O., Kuzovkov V.N. and von Niessen W. Phys.Rev.Lett. 1999, 83, p.3089-3092.



1.2.5. The Anderson localization

- **Kuzovkov V.N., von Niessen W., Kashcheyevs V. and Hein O.**
  J.Phys.: Condens. Matter., 2002, 14, p. 13777-13797.
- **Kuzovkov V.N., Kashcheyevs V. and von Niessen W.**
  J.Phys.: Condens. Matter., 2004, 16, p. 1683-1685.
- **Kuzovkov V.N. and von Niessen W.**
  Eur. Phys. J. B, 2004, 42, p. 529-542.
- **Kuzovkov V.N. and von Niessen W.**
  Physica A, 2006, 369, p. 251-265.
- **Kuzovkov V.N. and von Niessen W.**
  Physica A, 2007, 377, p. 115-124.
- **Kuzovkov V.N.**
  Phys. Stat. Sol.(b), 2009, 246, p. 1257-1267.

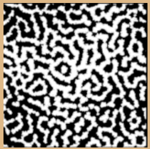


2. Fifty years of Anderson localization

2.1. Truth and mistake

B. Pascal

A truth is so delicate that any small deviation from it leads you to a mistake, but this mistake is also so delicate that after a small retreat you find yourselves in a truth again.



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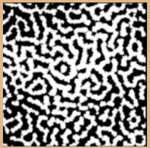
2.2. "Theorem of Anderson"

2.2.1. Anderson's 1958 paper

P.W. Anderson, *Absence of Diffusion in Certain Random Lattices*, Phys.Rev. 109, 1492 (1958)
Anderson's paper was cited just 30 times in the first 10 years. Today, it's been cited over 4000 times.

> *Very few believed [localization] at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author.*
> Philip. W. Anderson, Nobel lecture, 8 December 1977



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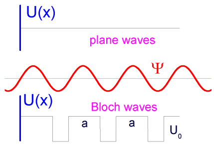
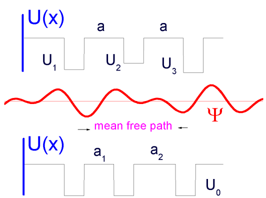
2.2.2. Experiment and Theory

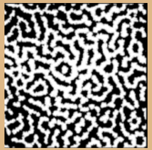
Anomalously long relaxation times of electron spins in doped semiconductors.
The concept of localized electrons could explain the observation but would break with the conventional (diffusion) picture.
Ideal crystal



The traditional view had been that scattering by the random potential causes the Bloch waves to lose phase coherence on the length scale of the mean free path. The wave function remains extended throughout the sample.



Typical wave functions of extended state with mean free path



2.2.3. Localized state

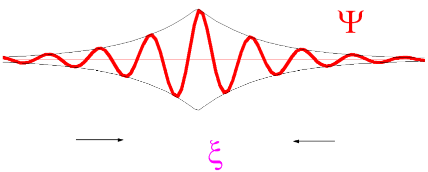
If the disorder is very strong, the wave function may become localized: the envelope of the wave function decays exponentially,

$$\Psi(r) \propto \exp(-r/\xi),$$

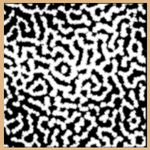
and ξ is the localization length.

- Extended state = metallic state
- Localized state = insulating state

Transition between a metal and an insulator = metal-insulator transition (MIT)



Typical wave functions of localized state with localization length ξ

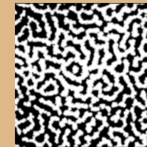


2.3. Scaling theory of localization

2.3.1. Scaling theory

Abrahams, Anderson, Licciardello and Ramakrishnan, Phys.Rev.Lett. 42, 673 (1979)

> *The years since the Nobel Prize (1977) have been productive ones for me. For instance, in 1978, shortly after receiving the prize in part for localization theory, I was one of the "Gang of Four" (with Elihu Abrahams, T.V. Ramakrishnan, and Don Licciardello) who revitalized that theory by developing a scaling theory which made it into a quantitative experimental science with precise predictions as a function of magnetic field, interactions, dimensionality, etc.; a major branch of science continues to flow from the consequences of this work.*
> - Philip Warren Anderson, Autobiography



2.3.2. Theoretical consensuns: no metallic behavior in two dimensions!

- All states are localized in one (1-D) dimensions.
- All states are localized in two[†] (2-D) dimensions.
- MIT is a continuous[†] one (second-order) in three (3-D) dimensions.

[†]Scaling theory of localization.

Critical dimensions:

- Ising-model, $D_c = 2$.
- Heisenberg-model, $D_c = 3$ (?).
- Anderson-model, $D_c = 3$.

L. Onsager and R. Feynman

> *Then Professor Onsager got up and said in a dour voice, "Well, Professor Feynman is new in our field, and I think he needs to be educated. There's something he ought to know, and we should tell him." I thought, "Geesus! What did I do wrong?" Onsager said, "We should tell Feynman that nobody has ever figured out the order of any transition correctly from first principles . . . "*
>
> - Richard P. Feynman, "Surely You're Joking, Mr Feynman!"

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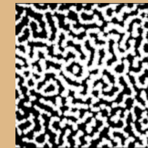
2.4. Experiment

2.4.1. Truth and mistake

L. Wittgenstein

The cognition process consists in the replacement of rough mistakes by more refined mistakes.



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2.4.2. Kravchenko and Co

Colloquium: Metallic behavior and related phenomena in two dimensions

> ***For about 20 years, it has been the prevailing view that there can be no metallic state or metal-insulator transition in two-dimensions in zero megnetic field. In the last several years, however, unusual behavior suggestive of such a transition has been reported in a variety of dilute two dimensional electron and hole systems. The physics behind these observations is at present not understood. The authors review and discuss the main experimental findings und suggested theoretical models.***
>
> Abstract, E. Abrahams, S.V.Kravchenko and M.P.Sarachik, Reviews of Modern Physics, 73, 251 (2001)

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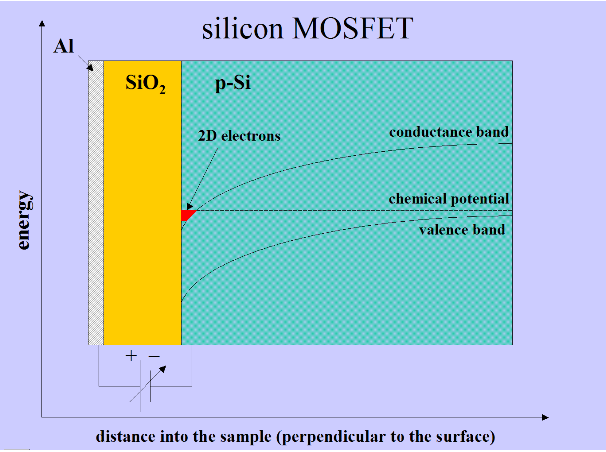
2.4.3. Si-MOSFET



In a 2-D system - such a very thin metal film, or active region of many semiconductor transistors - the electrons are constrained to move in a plane of negligible thickness.

> *Metal-insulator transition unexpectedly appears in a two-dimensional electron system.* -Physics Today, July 1997

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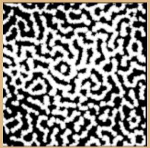
2.4.4. Economist, July 1997

2-D Metal-insulator Transition (1994)

- It is possible to create a narrow layer of electrons near the surface of the silicon crystal used to form what is known as a metal-oxide-semiconductor field effect transistor (MOSFET). This layer can conduct electricity.
- Theorists have proclaimed with great assurance: as such a two-dimensional electron gas is cooled, it freezes into an electrically insulating lattice.
- The discoverers of the "metal-insulator transition" (Kravchenko and his colleagues), however, believe the theorists are wrong.



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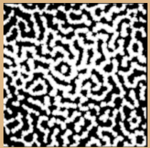
3. Deja vu

3.1. Experiment (Kravchenko and Co)

3.1.1. Truth and mistake

Georg Christoph Lichtenberg

The American who was the first to discover Columbus made a bad discovery.



Possible metal-insulator transition at $B = 0$ in two dimensions
S.V.Kravchenko et al,
Phys. Rev. B, 50, 8039 (1994)

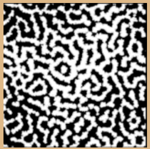
PHYSICAL REVIEW B VOLUME 50, NUMBER 11 15 SEPTEMBER 1994-I

Possible metal-insulator transition at $B = 0$ in two dimensions

S. V. Kravchenko, G. V. Kravchenko,* and J. E. Furneaux
Laboratory for Electronic Properties of Materials and Department of Physics and Astronomy, University of Oklahoma, Norman, Oklahoma 73019

V. M. Pudalov† and M. D'Iorio
National Research Council of Canada, Institute for Microstructural Science, Ottawa, Ontario, Canada K1A OR6
(Received 18 April 1994)

We have studied the zero magnetic field resistivity ρ of unique high-mobility two-dimensional electron systems in silicon. At very low electron density n_s (but higher than some sample-dependent critical value $n_{cr} \sim 10^{11}$ cm^{-2}), conventional weak localization is overpowered by a *sharp drop of ρ by an order of magnitude* with decreasing temperature below ~1–2 K. No further evidence for electron localization is seen down to at least 20 mK. For $n_s < n_{cr}$, the sample is insulating. The resistance is empirically found to scale with temperature both below and above n_{cr} with a single parameter that approaches zero at $n_s = n_{cr}$ suggesting a metal-insulator phase transition.



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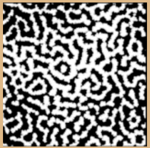
3.1.3. PRL 1994

Reaction of referees:

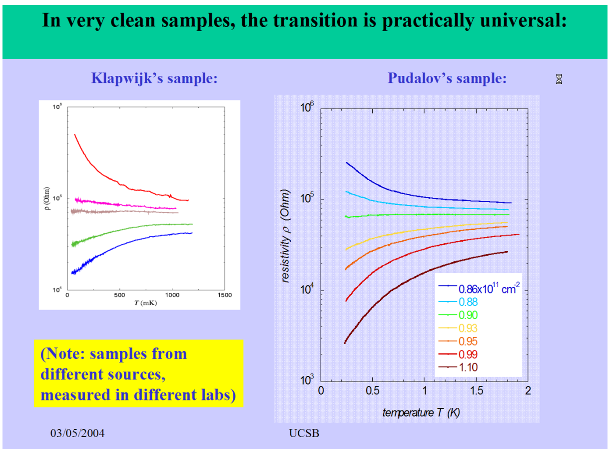
- The paper should not be published in PRL. Everyone knows there is no zero-temperature conductivity in 2-D.
- The results are not interesting. Silicon MOSFETs have been studied for 20 years...
- The reported results are most intriguing, but they must be wrong. If there indeed were a metal-insulator transition in these systems, it would have been noticed years ago.
- I do not see a physical mechanism behind the reported behavior. It must be an instrumental error.

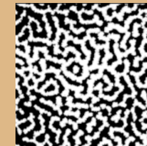


3.1.4. Power Point presentations 2003-2004

So...

- in experiment, only metallic behavior is seen in high-quality samples down to the lowest accessed temperatures.
- Then, why is it "metallic" or "so-called metallic" rather than just metallic?

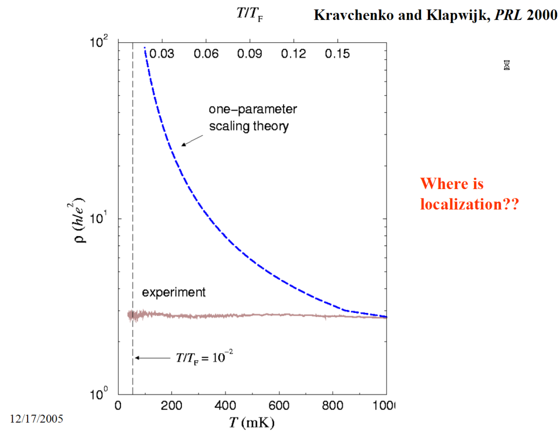




3.1.5. Power Point presentation 2003-2004: Discussion

So, once again...
Why is the state"so-called" metallic rather than just metallic?

- **pro: Experimental data in strongly interacting systems**
- **contra: Theory for weakly interacting systems**



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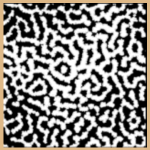
3.2. Theory (Kuzovkov and Co)

3.2.1. Truth and mistake

Georg Christoph Lichtenberg

The hypotheses of some innovators do not contradict our experience but I suspect that one day our experience will contradict them.



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3.2.2. JPCM 2002

Exact analytic solution for the generalized Lyapunov exponent of the two-dimensional Anderson localization
V.N. Kuzovkov et al,
J.Phys.: Condens. Matter, 14, 13777 (2002)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER
J. Phys.: Condens. Matter **14** (2002) 13777–13797 PII: S0953-8984(02)53297-5

Exact analytic solution for the generalized Lyapunov exponent of the two-dimensional Anderson localization

V N Kuzovkov[1,2], W von Niessen[2], V Kashcheyevs[1] and O Hein[2]

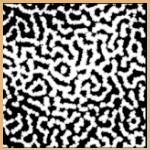
[1] Institute of Solid State Physics, University of Latvia, 8 Kengaraga Street, LV–1063 Riga, Latvia
[2] Institut für Physikalische und Theoretische Chemie, Technische Universität Braunschweig, Hans-Sommer-Straße 10, 38106 Braunschweig, Germany

E-mail: kuzovkov@latnet.lv

Received 11 September 2002
Published 6 December 2002

Results:

- $D_c = 2$.
- **MIT is a discontinuous one (first-order) in two (2-D) dimensions.**
- **Both extended and localized states may arise, and indeed may coexist in such a way that the localized states are not rendered extended (phase coexistence).**



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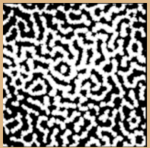
Referee A:

This is a most unusual paper, for in it the authors claim to provide an analytical solution to one of the major problems in condensed matter theory: two-dimensional Anderson localization in a disordered tight binding model.

- **If the present work is correct, it is highly important.**
- **Is it correct? I can only be honest and say I do not know.**
- **But if the work is correct, it will become a classic.**

Referee B:

- **If the conclusions of the paper are correct then much of our understanding of this problem will have to be revised.**



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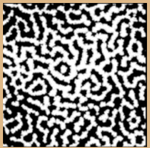
3.2.3. PRL 2001

Reaction of referees:

- The authors claim to have an exact calculation of the localization length in the two-dimensional Anderson model. If that were true, it would be an exciting result. However, admitting that I did not check all details of their derivation, after inspection of the starting point and of the result, I arrive at the conclusion that the author's claim is not justified.
- Although the result of the authors are not uninteresting, I feel that similar results must already exist in the old literature, and do NOT address the real problem that people have been trying to solve using much more sophisticated methods.
- The results of the paper are sensational. The authors claim to solve exactly the longstanding problem of the two-dimensional localization with highly unexpected results. If the results were correct, they would constitute a breakthrough compared only probably with Anderson's original paper. . . I think that the approach used is flawed, and that the conclusions of the paper are based upon a number of misunderstandings.



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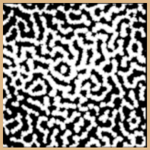
Final Referee

- It is the responsibility of the authors to explain why they get a different result from what is accepted and why their result should be the correct one.
- Although there is no general analytical solution availibel, there is consensuns that in two-dimensions all states are localized.
- The fact that certain experimental observations in Si-MOSFET's and other systems appear to suggest a metallic phase in a disordered, interacting 2-D electron system should not be used to argue that we do not understand disordered systems of noninteracting electrons.
- This is because if there is indeed a phase transition in these systems (which remains to be proven beyond doubt) it is most likely related to the Coulomb interaction or possibly the spin-orbit interaction.



3.3. Theoretical consensuns?

3.3.1. Truth and mistake

Georg Christoph Lichtenberg

This is a theory explaining the northern lights by the brilliance of the herring scales.

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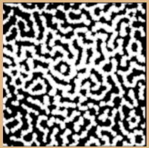
3.3.2. Analytic vs Numerical experiment

Analytic:

> *. . . Practically all numerical results on the critical behavior near the Anderson transition are in contradiction with the analytical predictions, but no serious discussion of this circumstance is given. In fact, a situation with numerical algorithms is rather serious . . .*
>
> - I.M. Suslov, arXiv:cond-mat/0610744 (2006)

Numerical experiment:

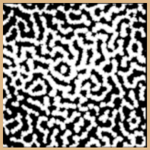
> *Disagreement between the theoretical predictions and numerical data might lead to the conclusion that the numerical scaling analysis is insufficient or even wrong. Our belief that the numerical data . . . are correct. All mean values can be calculated "from first principles". In our opinion, the discrepancy between the numerical data and results of analytical theories is due to inability of analytical theories to analyze completely the statistical fluctuations in the critical regime.*
>
> - P. Markoš, Acta physica slovaca, 56, 561 (2006)



3.3.3. Perpetuum mobile

Reaction of referees:

- . . . the authors have oversold their results and drawn unwarranted conclusions from them. I am therefore unable to recommend publication of this paper in its current form.
- Most of the results are in contradiction to well accepted general results of the scaling theory and of numerical investigations. . . The paper will certainly stimulate further investigations and discussions. Therefore I recommend publication.
- This paper does not present any new results . . . Therefore, I cannot recommend this paper for publication.
- One surprising result is the coexistence of localized and extended states for the same parameter values. . . I recommend publication.
- . . . authors seem to ignore most of the extensive literature that has been published on the subject.



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4. Anderson localization and the generalized diffusion approach

4.1. (Analytical) theory vs (numerical) experiment

4.1.1. Cauchy problem

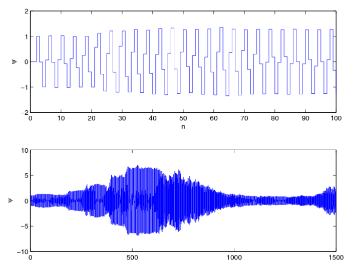
To explain the effect, Anderson used a tight-binding model of an electron in a disordered lattice; at each lattice site an electron feels a random potential, ε_n, and is allowed to tunnel between nearest neighbor sites with a constant rate.

$$\psi_{n+1} + \psi_{n-1} = (E - \varepsilon_n)\psi_n.$$

Chaos theory: The Schrödinger equation for the 1-D Anderson model as a recursive relation (n - discrete time):

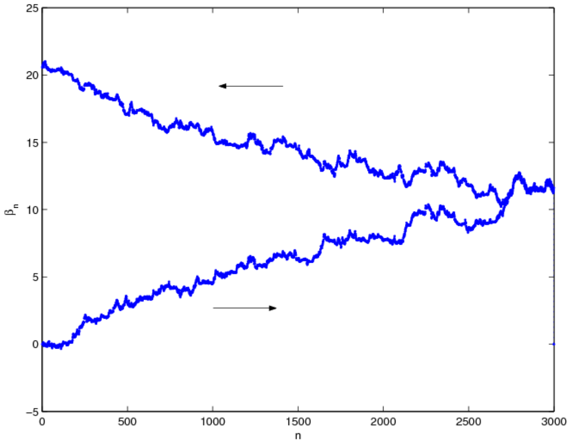
$$\psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}.$$

The Cauchy problem with the *fixed initial conditions* for ψ_1 and ψ_0.



4.1.2. Reversibility and Numerical instability

$$\Longrightarrow \psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}.$$
$$\Longleftarrow \psi_{n-1} = (E - \varepsilon_n)\psi_n - \psi_{n+1}.$$



$$\beta_n = \frac{1}{2}\ln(\psi_{n+1}^2 + \psi_n^2)$$

4.2. Anderson localization and classical Hamiltonian map

4.2.1. Classical Hamiltonian map for one-dimensional case

Classical phase space (p, q)
[F.M. Izrailev and A.A. Krokhin, Phys.Rev.Lett., 82, 4062 (1999)]
The representation of quantum tight-binding model with diagonal disorder

$$\psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}.$$

in terms of classical two-dimensional Hamiltonian map.
Definitions: $q_n = sign(E)\psi_n$, $p_n = sign(E)\left(\psi_{n+1} - \psi_n\right)$

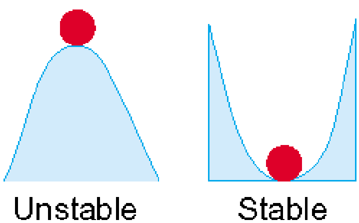
$$p_{n+1} = p_n - \omega^2 q_n - \varepsilon_n q_n,$$
$$q_{n+1} = q_n + p_{n+1}.$$

Kicked-oscillator Hamiltonian

$$\mathcal{H} = \frac{p^2}{2} + \frac{\omega^2 q^2}{2} + \frac{\varepsilon(t) q^2}{2} \sum_n \delta(t - n\Delta t).$$

$\omega^2 = 2 - \lfloor E \rfloor$.

It defines the system with unperturbed Hamiltonian of oscillator which is affected by a periodic sequence of kicks (δ-pulses) with the period Δt.



- $\lfloor E \rfloor > 2$ **(outside the band) - unstable mode**
- $\lfloor E \rfloor < 2$ **(inside the band) - stable mode**

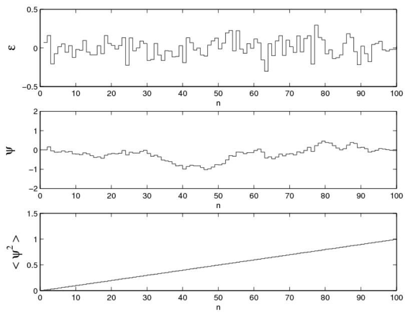
4.2.3. Diffusion and divergence

Random walks and normal diffusion.

$$\psi_{n+1} = \psi_n + \varepsilon_n.$$

$$\langle \psi_n^2 \rangle = \langle \psi_0^2 \rangle + \sigma^2 n.$$

$\sigma^2 = \langle \varepsilon_n^2 \rangle$



The diffusion motion is characterized by divergences, e.g. for the mean time when the system returns to the initial state.
To detect the diffusion, it is *sufficient* to demonstrate the divergence of the second moment of the amplitude and to establish its law of time-dependence.

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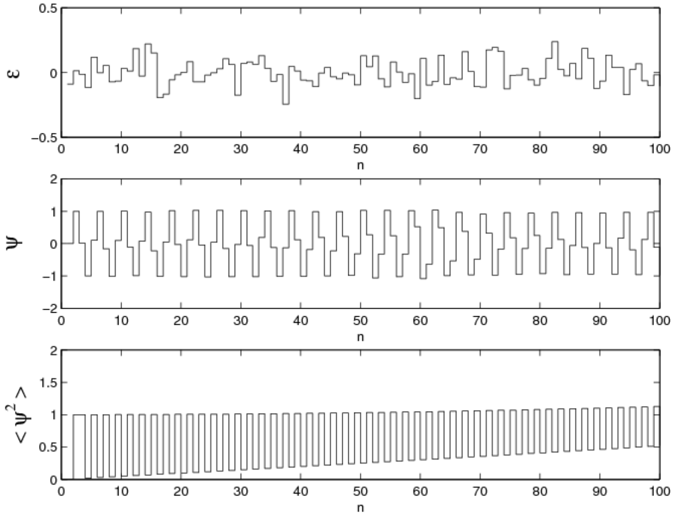
4.2.4. Localization as generalized diffusion



Lyapunov exponent:

$$\langle \psi_n^2 \rangle \propto \exp(2\gamma n).$$

Localization length:
$\xi = 1/\gamma$.

4.3. Analytical theory

4.3.1. Causality principle

[L. Molinari. J.Phys.A: Math. Gen. 25, 513 (1992)]

$$\psi_{n+1} = (E - \varepsilon_n)\psi_n - \psi_{n-1}.$$

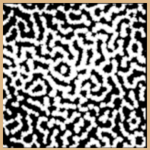
Is is easy to see that:

- ψ_2 **is a function of** ε_1**,**
- ψ_3 **is a function of** ε_2**,**ε_1**,**
- ψ_{n+1} **is a function of** $\varepsilon_n, \ldots, \varepsilon_2$**,**$\varepsilon_1$ **(causality).**

So both amplitudes ψ_n **and** ψ_{n-1} **on the rhs of equation are statistical independent of** ε_n **and can be averaged separately (causality principle):**

$$\langle \psi_{n+1} \rangle = E\langle \psi_n \rangle - \langle \psi_{n-1} \rangle,$$
$$\langle \psi_{n+1}^2 \rangle = (E^2 + \langle \varepsilon_n^2 \rangle)\langle \psi_n^2 \rangle - 2E\langle \psi_n \psi_{n-1} \rangle + \langle \psi_{n-1}^2 \rangle,$$

etc.



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4.3.2. Cauchy problem for the Schrödinger equation in N-D

The semi-infinite system, or infinite system with a boundary, where the index $n \equiv m_D \geq 0$, but all $m_j \in (-\infty, \infty)$, $j = 1, 2, \ldots, p$, with

$$p = D - 1.$$

We combine indices in the vector $\mathbf{m} = \{m_1, m_2, \ldots, m_p\}$. The boundary which is the layer $n = 0$ defines the preferential direction (the axis n).

$$\psi_{n+1,\mathbf{m}} = (E - \varepsilon_{n,\mathbf{m}})\psi_{n,\mathbf{m}} - \psi_{n-1,\mathbf{m}} - \sum_{\mathbf{m}'} \psi_{n,\mathbf{m}'}.$$

Summation over $\mathbf{m}'$ includes the nearest neighbours of the site $\mathbf{m}$. The on-site potentials $\varepsilon_{n,m}$ are independently and identically distributed. We assume hereafter existence of the two first moments,

$$\langle \varepsilon_{n,\mathbf{m}} \rangle = 0,$$
$$\langle \varepsilon^2_{n,\mathbf{m}} \rangle = \sigma^2,$$

where the parameter σ characterizes the disorder level.

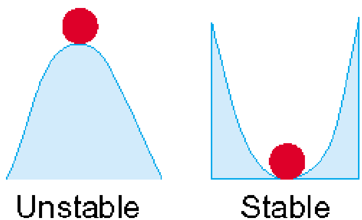
4.3.3. Fourier transform

$$\varepsilon_n(\mathbf{k}) = \sum_{\mathbf{m}} \varepsilon_{n,\mathbf{m}} e^{i\mathbf{k}\mathbf{m}},$$
$$\psi_n(\mathbf{k}) = \sum_{\mathbf{m}} \psi_{n,\mathbf{m}} e^{i\mathbf{k}\mathbf{m}}.$$

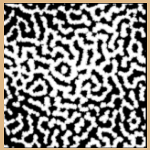
Schrödinger equation

$$\psi_{n+1}(\mathbf{k}) = \mathcal{L}(\mathbf{k})\psi_n(\mathbf{k}) - \psi_{n-1}(\mathbf{k}) - \int \frac{d^p\mathbf{k}_1}{(2\pi)^p} \varepsilon_n(\mathbf{k}-\mathbf{k}_1)\psi_n(\mathbf{k}_1).$$

$$\mathcal{L}(\mathbf{k}) = E - 2\sum_{j=1}^{p=D-1} \cos(k_j),$$



- $\lfloor \mathcal{L}(\mathbf{k}) \rfloor > 2$ **(outside the band) - unstable mode**
- $\lfloor \mathcal{L}(\mathbf{k}) \rfloor < 2$ **(inside the band) - stable mode**



4.3.4. Diffusion and localization

- **Initial conditions - ONLY stable modes**
- **Bounded first moment** $\lfloor\langle\psi_n(\mathbf{k})\rangle\rfloor < \infty$ **for** $n \to \infty$
- **Delocalized states - bounded second moment,** $\langle Q_n^2\rangle < \infty$
- **Localized states - unbounded second moment,** $\langle Q_n^2\rangle \to \infty$ **for** $n \to \infty$

Q^2-Definition

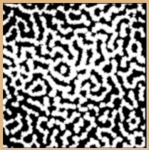
$$Q_n^2 \equiv U_n = \int \frac{d^p\mathbf{k}}{(2\pi)^p}\langle\lfloor\psi_n(\mathbf{k})\rfloor^2\rangle.$$

Unperturbed system ($\sigma = 0$):

$$U_n^{(0)} < \infty \; for \, n \to \infty.$$

Z-transform:

$$U(z) = \sum_{n=1}^{\infty} \frac{U_n}{z^n}.$$



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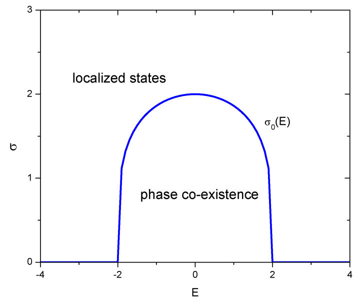
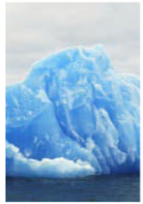
4.3.5. Localization operator $H(z)$ and phase diagram

$$\frac{1}{H(z)} = 1 - \sigma^2 \frac{(z+1)}{(z-1)} \int \frac{d^p\mathbf{k}}{(2\pi)^p} \frac{1}{[(z+1)^2/z - \mathcal{L}^2(\mathbf{k})]}.$$

The concept of the localization operator $H(z)$ is a general and abstract description of the problem of localization. Instead of analyzing wave functions, it is sufficient to analyse properties of $H(z)$ by means of the theory for functions of complex variables.
Knowledge of the fundamental characteristics of the system - the localization operator $H(z)$ - permits to determine the phase diagram of the system (regions of localized and delocalized solutions), as well as the localization length.

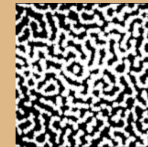


4.4. New theory and experiment



The metal-insulator transition should be looked at from the basis of first-order phase transition theory. This opinion differs from the traditional point of view, which considers this transition as continuous (second-order).

> *A single electron transistor is used as a local electrostatic probe to study the underlying spatial structure of the metal-insulator transition in two-dimensions. The measurement show that as we approach the transition from the metallic side, a new phase emerges that consists of weakly coupled fragments of the two-dimensional system. These fragments consist of localized charge that coexists with the surrounding metallic phase. As the density is lowered into the insulating phase, the number of fragments increases on account of the disappearing metallic phase.*
>
> -Abstract,
> S.Ilani, A.Yacoby, D.Mahalu and H.Shtrikman, Science, 292, 1354 (2001)



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4.5. Truth and mistake

M. Planck's Principle

A new scientific truth does not triumph because its supporters enlighten its opponents, but because its opponents eventually die, and a new generation grows up that is familiar with it.